9.4.2

Logical Inference 2

Rule-based reasoning

Chapter 9

Some material adopted from notes by Andreas Geyer-Schulz,, Chuck Dyer, and Mary Getoor

Automated inference for FOL

- Automated inference for FOL is harder than PL
  - Variables can take on an infinite number of possible values from their domains
  - Hence there are potentially an infinite number of ways to apply the Universal Elimination rule
- Godel's Completeness Theorem says that FOL entailment is only semi-decidable
  - If a sentence is true given a set of axioms, there is a procedure that will eventually determine this
  - If a sentence is false, there's no guarantee a procedure will ever discover this — it may never halt

Generalized Modus Ponens (GMP)

- Modus Ponens: **P, P=>Q |= Q**
- Generalized Modus Ponens extends this to rules in FOL
- Combines And-Introduction, Universal-Elimination, and Modus Ponens, e.g.
  - given P(c) , Q(c) , $\forall$x P(x)$\wedge$Q(x) $\rightarrow$ R(x)
  - derive R(c)
- Must deal with
  - more than one condition on rule's left side
  - variables

Often rules restricted to Horn clauses

- A Horn clause is a sentence of the form:

  $P_1(x) \wedge P_2(x) \wedge \ldots \wedge P_n(x) \rightarrow Q(x)$

  where
  - ≥ 0 P_is and 0 or 1 Q
  - P_is and Q are positive (i.e., non-negated) literals
- Prolog and most rule-based systems are limited to Horn clauses
- Horn clauses are a subset of all FOL sentences

Horn clauses 2

- Special cases
  - Typical rule: $P_1 \wedge P_2 \wedge \dots P_n \rightarrow Q$
  - Constraint: $P_1 \wedge P_2 \wedge \dots P_n \rightarrow$ false
  - A fact: $\rightarrow Q$
  - A goal: $Q \rightarrow$
- Examples
  - parent(P1,P2) $\wedge$ parent(P2,P3) $\rightarrow$ grandparent(P1,P3)
  - male(X) $\wedge$ female(X) $\rightarrow$ false
  - $\rightarrow$ male(john)
  - female(mary) $\rightarrow$

Horn clauses 3

- These are not Horn clauses:
  - married(x, y) $\rightarrow$ loves(x, y) $\vee$ hates(x, y)
  - $\neg$ likes(john, mary)
  - $\neg$ likes(x, y) $\rightarrow$ hates(x, y)
- Can’t assert/conclude disjunctions (i.e., an ”or”)
- Can’t have “true” negation
  - Though some systems, like Prolog, allow a negation operator that means “can’t prove”
- No wonder Horn clause reasoning is easier

Horn clauses 3

- Where are the quantifiers?
  - Variables in conclusion universally quantified
  - Variables only appearing in premises existentially quantified
- Examples:
  - $parentOf(P,C) \rightarrow childOf(C,P)$
    $\forall P\ \forall C\ parentOf(P,C) \rightarrow childOf(C,P)$
  - $parentOf(P,X) \rightarrow isParent(P)$
    $\forall P\ \exists X\ parent(P,X) \rightarrow isParent(P)$
  - $parent(P1, X) \wedge parent(X, P2) \rightarrow grandParent(P1, P2)$
    $\forall P1,P2\ \exists X\ parent(P1,X) \wedge parent(X, P2)$
    $\rightarrow grandParent(P1, P2)$

Definite Clauses

- A **definite clause** is a horn clause with a conclusion
- What's not allowed is a horn clause w/o a conclusion, e.g.
  - male(x), female(x) $\rightarrow$
  - i.e., ~male(x) $\vee$ ~female(x)
- Most rule-based reasoning systems, like Prolog, allow only definite clauses in the KB

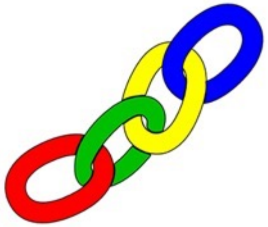
Limitations

- Most rule-based reasoning systems use only definite horn clauses
  - Limited ability to reason about negation and disjunction
- Benefit is decidability and efficiency
- Some limitations can be overcome by
  - Adding procedural components
  - Augmenting with other reasoners

Forward & Backward Reasoning

- We often talk about two reasoning strategies:
  - Forward chaining and
  - Backward chaining
- Both are equally powerful, but optimized for different use cases
- You can also have a mixed strategy

Forward chaining

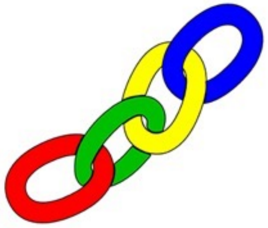


- Proofs start with given axioms/premises in KB, deriving new sentences using GMP until the goal/query sentence is derived
  - Process follows a chain of rules and facts going from the KB to the conclusion
- This defines a forward-chaining inference procedure because it moves “forward” from the KB to the goal [eventually]
- Inference using GMP is sound and complete for KBs containing only Horn clauses

Forward chaining example

- KB:
  - allergies(X) $\rightarrow$ sneeze(X)
  - cat(Y) $\wedge$ allergicToCats(X) $\rightarrow$ allergies(X)
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Backward chaining



- Backward-chaining deduction using GMP is also complete for KBs containing only Horn clauses
- Proofs start with the goal query, find rules with that conclusion, and then tries to prove each of the antecedents in the rule
- Keep going until you reach premises
- Avoid loops by checking if new subgoal is already on the goal stack
- Avoid repeated work: use a cache to check if new subgoal already proved true or failed

Backward chaining example

- KB:
  - allergies(X) $\rightarrow$ sneeze(X)
  - cat(Y) $\wedge$ allergicToCats(X) $\rightarrow$ allergies(X)
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Forward vs. backward chaining

- Forward chaining is data-driven
  - Automatic, unconscious processing, e.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
  - Efficient when you want to compute **all conclusions**
- Backward chaining is goal-driven, better for problem-solving and query answering
  - Where are my keys? How do I get to my next class?
  - Complexity can be much less than linear wrt KB size
  - Efficient when you want **one or a few conclusions**
  - Good where the underlying facts are changing

Mixed strategy

- Many practical reasoning systems do both forward and backward chaining
- How you encode rule determines how it's used:

  spouse(X,Y) => spouse(Y,X)   % forward chaining

  father(X,Y) <= parent(X,Y), male(X)   % backward chaining
- Forward chaining rules useful if you want to draw conclusions and **materialize them as facts**
  - This also easily avoid loops, e.g., don't trigger forward chaining if the fact already exists
- Given a model of your rules and the kind of reasoning needed, you can decide which to encode as FC and which as BC rules

Completeness of GMP

- GMP (using forward or backward chaining) is complete for KBs that contain only Horn clauses
- not complete for simple KBs with non-Horn clauses
- What is entailed by the following sentences:

1. $(\forall x)\ P(x) \rightarrow Q(x)$
2. $(\forall x)\ \neg P(x) \rightarrow R(x)$
3. $(\forall x)\ Q(x) \rightarrow S(x)$
4. $(\forall x)\ R(x) \rightarrow S(x)$

Completeness of GMP

- The following entail that S(A) is true:
    1. $(\forall x)\ P(x) \rightarrow Q(x)$
    2. $(\forall x)\ \neg P(x) \rightarrow R(x)$
    3. $(\forall x)\ Q(x) \rightarrow S(x)$
    4. $(\forall x)\ R(x) \rightarrow S(x)$
- If we want to conclude S(A), with GMP we cannot, since the second one is not a Horn clause
- It is equivalent to $P(x) \vee R(x)$

Fin