

Propositional Logic

Chapter 7.4–7.7

Motivation



- Enabling the Wumpus World agent to use **propositional logic** allows her to reason about what she observes
 - to infer additional facts that must be true
 - to identify those facts that must be false
- Increasing her chances of finding the gold and avoiding the Wumpus and pits
- Propositional logic is simple and efficient

Propositional logic syntax

- Users specify
 - Set of propositional symbols (e.g., P, Q) whose values can be **True** or **False**
 - What each *means*, e.g.: P: “*It’s hot*”, Q: “*It’s humid*”
- A sentence (well formed formula) is defined as:
 - Any symbol is a sentence
 - If S is a sentence, then $\neg S$ is a sentence
 - If S is a sentence, then **(S)** is a sentence
 - If S and T are sentences, then so are **(S \vee T)**, **(S \wedge T)**, **(S \rightarrow T)**, and **(S \leftrightarrow T)**
 - A finite number of applications of the rules

Examples of PL sentences

- Q

“It’s humid”

- $Q \rightarrow P$

“If it’s humid, then it’s hot”

- $(P \wedge Q) \rightarrow R$

“If it’s hot and it’s humid, then it's raining”

- We’re free to choose better symbols, e.g.:

Hot for “It’s hot”

Humid for “It’s humid”

Raining for “It’s raining”

Some terms

- Given the truth values of all symbols in a sentence, it can be *evaluated* to determine its **truth value** (True or False)
- We consider a **Knowledge Base** (KB) to be a set of sentences that are all True
- A **model** for a KB is a **possible world** – an assignment of truth values to propositional symbols that makes each KB sentence true
 - There might be more than one way to do this

A simple example

Models for the KB

The KB

P is True
Q is True or
R is False

P
$Q \vee \neg R$

P	Q	R	
T	T	F	#1
T	T	T	#2
T	F	F	#3

The KB has 2
sentences

The KB has 3
variables

- Eight (2^3) ways to assign T/F to the three variables
- But the KB has just **three models**
- Each model has a value for every variable in the KB such that every sentence is true
- These are three possible worlds consistent with the KB

Another simple example

The KB

P is True and
Q is True

R is True and
P is False

$$\begin{aligned} P \wedge Q \\ R \wedge \neg P \end{aligned}$$

The KB has 2
sentences

The KB has 3
variables

Models for the KB

P	Q	R
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- The KB has **no models**
- There's no assignment of True or False to every variable that makes every KB sentence true
- There's no possible world consistent with the KB
- The KB is **inconsistent**

More terms

- **valid sentence** or **tautology**: one that's **True** under all interpretations, no matter what the world is actually like or what the semantics is. Example: “It's raining or it's not raining” ($P \vee \neg P$)
- An **inconsistent sentence** or **contradiction**: a sentence that's **False** under all interpretations. The world is never like what it describes, as in “It's raining and it's not raining.” ($P \wedge \neg P$)

Truth tables

Used to define meaning of logical connectives

Truth tables for the five logical connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>False</i>	<i>False</i>	<i>True</i>	<i>False</i>	<i>False</i>	<i>True</i>	<i>True</i>
<i>False</i>	<i>True</i>	<i>True</i>	<i>False</i>	<i>True</i>	<i>True</i>	<i>False</i>
<i>True</i>	<i>False</i>	<i>False</i>	<i>False</i>	<i>True</i>	<i>False</i>	<i>False</i>
<i>True</i>	<i>True</i>	<i>False</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>

e.g., Given a value for P and for Q, the truth table **defines** the value of $P \vee Q$

Truth tables

Used to define meaning of logical connectives and to determine when a complex sentence is true given values of its symbols

Truth tables for the five logical connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \equiv Q$
False	False	True	False	False	True
False	True	True	False	True	False
True	False	False	False	True	False
True	True	False	True	True	True

The value of complex sentences can be determined from the values of their elements

Example of a truth table used for a complex sentence

P	H	$P \vee H$	$(P \vee H) \wedge \neg H$	$((P \vee H) \wedge \neg H) \Rightarrow P$
False	False	False	False	True
False	True	True	False	True
True	False	True	True	True
True	True	True	False	True

Implies connective: $P \rightarrow Q$

\rightarrow is a *logical connective* (not a “rule”)

- $P \rightarrow Q$ is a **logical sentence** and has a truth value, i.e., is either **True** or **False**
- If the sentence is in a KB, it can be used by a rule (*Modes Ponens*) to infer that **Q is True** if **P is True** in the KB
- Given a KB where $P = \text{True}$ and $Q = \text{True}$, we can derive/infer/prove that $P \rightarrow Q$ is True
- Note: $P \rightarrow Q$ is equivalent to $\sim P \vee Q$

$$P \rightarrow Q$$

When is $P \rightarrow Q$ true? Check all that apply

- $P=Q=\text{true}$
- $P=Q=\text{false}$
- $P=\text{true}, Q=\text{false}$
- $P=\text{false}, Q=\text{true}$

$$P \rightarrow Q$$

When is $P \rightarrow Q$ true? Check all that apply

$P=Q=\text{true}$

$P=Q=\text{false}$

$P=\text{true}, Q=\text{false}$

$P=\text{false}, Q=\text{true}$

- We can get this from the truth table for \rightarrow
- Note: in FOL it's much harder to prove that a conditional true, e.g., $\text{prime}(x) \rightarrow \text{odd}(x)$
you must prove it's true for every possible value of x

$$P \rightarrow Q$$

When is $P \rightarrow Q$ true? Check all that apply

- $P=Q=\text{true}$
- $P=Q=\text{false}$
- $P=\text{true}, Q=\text{false}$
- $P=\text{false}, Q=\text{true}$

If it helps, think of $P \rightarrow Q$ as meaning

Whenever P is True, Q has to be True;

When P is False, it say nothing about Q .

$$P \rightarrow Q \equiv \sim P \vee Q$$

- $P \rightarrow Q$ is equivalent to $\sim P \vee Q$
- We can show this by looking at a truth table

P	Q	$P \rightarrow Q$	$\sim P \vee Q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

**These two columns
are the same**

Models for a KB

- KB: $[P \vee Q, P \rightarrow R, Q \rightarrow R]$
- What are the **sentences**?
 - s1: $P \vee Q$
 - s2: $P \rightarrow R$
 - s3: $Q \rightarrow R$
- What are the propositional **variables**?
P, Q, R
- What are the candidate **models**?
 - 1) Consider all **eight** possible assignments of T|F to P, Q, R
 - 2) Check if each sentence is consistent with the model

P	Q	R	s1	s2	s3
F	F	F	X	✓	✓
F	F	T	X	✓	✓
F	T	F	✓	✓	X
F	T	T	✓	✓	✓
T	F	F	✓	X	✓
T	F	T	✓	✓	✓
T	T	F	✓	X	X
T	T	T	✓	✓	✓

Here **X** means the model makes the sentence False and **✓** means it doesn't make it False

Models for a KB

- KB: $[P \vee Q, P \rightarrow R, Q \rightarrow R]$
- What are the **sentences**?
 - s1: $P \vee Q$
 - s2: $P \rightarrow R$
 - s3: $Q \rightarrow R$
- What are the propositional **variables**?
 - P, Q, R
- What are the candidate **models**?
 - 1) Consider all possible assignments of T|F to P, Q, R
 - 2) Check truth tables for consistency, eliminate a row that does not make every KB sentence true

P	Q	R	s1	s2	s3
F	F	F	X	✓	✓
F	F	T	X	✓	✓
F	T	F	✓	✓	X
F	T	T	✓	✓	✓
T	F	F	✓	X	✓
T	F	T	✓	✓	✓
T	T	F	✓	X	X
T	T	T	✓	✓	✓

- Only 3 models are **consistent** with **KB**
- R true in **all** of them
- \therefore R is true and can be added to KB
- **We infer that R is true**

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