9.4.2

Logical Inference

Rule-based reasoning

Chapter 9

Some material adopted from notes by Andreas Geyer-Schulz,, Chuck Dyer, and Mary Getoor

Automated inference for FOL

- Automated inference for FOL is harder than PL
  - Variables can take on an infinite number of possible values from their domains
  - Hence there are potentially an infinite number of ways to apply the Universal Elimination rule
- Godel's Completeness Theorem says that FOL entailment is only semi-decidable
  - If a sentence is true given a set of axioms, there is a procedure that will determine this
  - If a sentence is false, there’s no guarantee a procedure will ever discover this — it may never halt

Generalized Modus Ponens (GMP)

- Modus Ponens: **P, P=>Q |= Q**
- Generalized Modus Ponens extends this to rules in FOL
- Combines And-Introduction, Universal-Elimination, and Modus Ponens, e.g.
  - given P(c) , Q(c) , $\forall$x P(x)$\wedge$Q(x) $\rightarrow$ R(x)
  - derive R(c)
- Must deal with
  - more than one condition on rule's left side
  - variables

Often rules restricted to Horn clauses

- A Horn clause is a sentence of the form:

  $P_1(x) \wedge P_2(x) \wedge ... \wedge P_n(x) \rightarrow Q(x)$

  where
  - ≥ 0 P_is and 0 or 1 Q
  - P_is and Q are positive (i.e., non-negated) literals
- Equivalently: $P_1(x) \vee P_2(x) ... \vee P_n(x)$ where P_i are all atomic and **at most one** is positive
- Prolog is based on Horn clauses
- Horn clauses are a subset of all sentences representable in FOL

Horn clauses 2

- Special cases
  - Typical rule: $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow Q$
  - Constraint: $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow$ false
  - A fact: $\rightarrow Q$
  - A goal: $Q \rightarrow$
- These are not Horn clauses:
  - married(x, y) $\rightarrow$ loves(x, y) $\vee$ hates(x, y)
  - $\neg$likes(john, mary)
  - $\neg$likes(x, y) $\rightarrow$ hates(x, y)
- Can't assert/conclude disjunctions, no negation
- No wonder reasoning over Horn clauses is easier

Horn clauses 3

- Where are the quantifiers?
  - Variables in conclusion universally quantified
  - Variables only appearing in premises existentially quantified
- Examples:
  - parentOf(P,C) $\rightarrow$ childOf(C,P)
    $\forall$P $\forall$C parentOf(P,C) $\rightarrow$ childOf(C,P)
  - parentOf(P,X) $\rightarrow$ isParent(P)
    $\forall$P $\exists$X parent(P,X) $\rightarrow$ isParent(P)
  - parent(P1, X) $\wedge$ parent(X, P2) $\rightarrow$ grandParent(P1, P2)
    $\forall$P1,P2 $\exists$X parent(P1,X) $\wedge$ parent(X, P2)
    $\rightarrow$ grandParent(P1, P2)

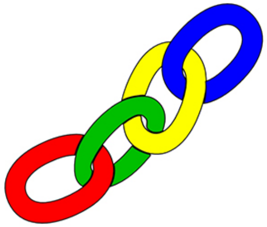
Definite Clauses

- A **definite clause** is a horn clause with a conclusion
- What's not allowed is a horn clause w/o a conclusion, e.g.
  - male(x), female(x) $\rightarrow$
  - i.e., male(x) $\vee$ female(x)
- Most rule-based reasoning systems, like Prolog, allow only definite clauses in the KB

Forward & Backward Reasoning

- We often talk about two reasoning strategies:
  - Forward chaining and
  - Backward chaining
- Both are equally powerful, but optimized for different use cases
- You can also have a mixed strategy

Forward chaining

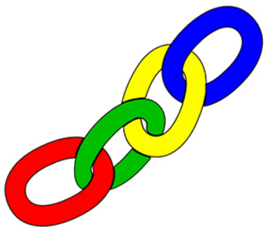


- Proofs start with given axioms/premises in KB, deriving new sentences using GMP until the goal/query sentence is derived
  - The process follows a chain of rules and facts going from the KB to the conclusion
- This defines a forward-chaining inference procedure because it moves "forward" from the KB to the goal [eventually]
- Inference using GMP is sound and complete for KBs containing only Horn clauses

Forward chaining example

- KB:
  - $\text{allergies}(X) \rightarrow \text{sneeze}(X)$
  - $\text{cat}(Y) \wedge \text{allergicToCats}(X) \rightarrow \text{allergies}(X)$
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Backward chaining



- Backward-chaining deduction using GMP is also complete for KBs containing only Horn clauses
- Proofs start with the goal query, find rules with that conclusion, and then tries to prove each of the antecedents in the rule
- Keep going until you reach premises
- Avoid loops by checking if new subgoal is already on the goal stack
- Avoid repeated work: use a cache to check if new subgoal already proved true or failed

Backward chaining example

- KB:
  - allergies(X) $\rightarrow$ sneeze(X)
  - cat(Y) $\wedge$ allergicToCats(X) $\rightarrow$ allergies(X)
  - cat(felix)
  - allergicToCats(mary)
- Goal:
  - sneeze(mary)

Forward vs. backward chaining

- Forward chaining is data-driven
  - Automatic, unconscious processing, e.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
  - Efficient when you want to compute all conclusions
- Backward chaining is goal-driven, better for problem-solving and query answering
  - Where are my keys? How do I get to my next class?
  - Complexity can be much less than linear w.r.t KB size
  - Efficient when you want one or a few decisions
  - Good where the underlying facts are changing

Mixed strategy

- Many practical reasoning systems do both forward and backward chaining
- The way you encode a rule determines how it is used, as in

  % this is a forward chaining rule

  spouse(X,Y) => spouse(Y,X).

  % this is a backward chaining rule

  wife(X,Y) <= spouse(X,Y), female(X).

- Given a model of the rules you have and the kind of reason you need to do, it's possible to decide which to encode as FC and which as BC rules.

Completeness of GMP

- GMP (using forward or backward chaining) is complete for KBs that contain only Horn clauses

- not complete for simple KBs with non-Horn clauses

- What is entailed by the following sentences:

1. $(\forall x)\ P(x) \rightarrow Q(x)$
2. $(\forall x)\ \neg P(x) \rightarrow R(x)$
3. $(\forall x)\ Q(x) \rightarrow S(x)$
4. $(\forall x)\ R(x) \rightarrow S(x)$

Completeness of GMP

- The following entail that S(A) is true:
  1. $(\forall x)\ P(x) \rightarrow Q(x)$
  2. $(\forall x)\ \neg P(x) \rightarrow R(x)$
  3. $(\forall x)\ Q(x) \rightarrow S(x)$
  4. $(\forall x)\ R(x) \rightarrow S(x)$
- If we want to conclude S(A), with GMP we cannot, since the second one is not a Horn clause
- It is equivalent to $P(x) \vee R(x)$