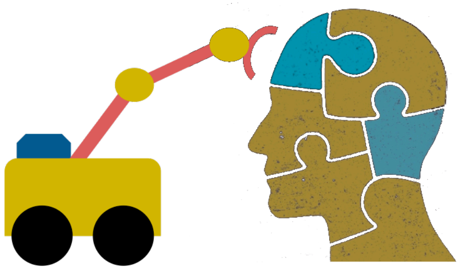
9.3.2



First-Order Logic (FOL) part 2

Overview

- We’ll first give some examples of how to translate between FOL and English
- Then look at modelling family relations in FOL
- And finally touch on a few other topics

Translating English to FOL

Every gardener likes the sun

$\forall x\ gardener(x) \rightarrow likes(x,Sun)$

All purple mushrooms are poisonous

$\forall x\ (mushroom(x) \land purple(x)) \rightarrow poisonous(x)$

No purple mushroom is poisonous (two ways)

$\neg\exists x\ purple(x) \land mushroom(x) \land poisonous(x)$

$\forall x\ (mushroom(x) \land purple(x)) \rightarrow \neg poisonous(x)$

English to FOL: Counting

Use = predicate to identify different individuals

- **There are <u>at least</u> two purple mushrooms**

  $\exists x\ \exists y\ \text{mushroom}(x) \land \text{purple}(x) \land \text{mushroom}(y) \land \text{purple}(y) \land \neg\mathbf{(x{=}y)}$

- **There are <u>exactly</u> two purple mushrooms**

  $\exists x\ \exists y\ \text{mushroom}(x) \land \text{purple}(x) \land \text{mushroom}(y) \land \text{purple}(y) \land \neg\mathbf{(x{=}y)} \land$
  $\forall z\ (\text{mushroom}(z) \land \text{purple}(z)) \to \mathbf{((x{=}z) \lor (y{=}z))}$

Saying there are 802 different Pokemon is hard!

Direct use of FOL is not for everything!

Translating English to FOL

What do these mean?

- You can fool *some of* the people *all of* the time
- You can fool *all of* the people *some of* the time

Translating English to FOL

What do these mean?

Both English statements are ambiguous

- **You can fool *some of* the people *all of* the time**

  There is a nonempty subset of people so easily fooled that you can fool that subset every time*

  For any given time, there is a non-empty subset at that time that you can fool

- **You can fool *all of* the people *some of* the time**

  There are one or more times when it's possible to fool everyone*

  Each individual can be fooled at some point in time

* Most common interpretation, I think

Some terms we will need

- **person(x):** True iff x is a person
- **time(t):** True iff t is a point in time
- **canFool(x, t):** True iff x can be fooled at time t

Note: *iff* = *if and only if* = ↔

Translating English to FOL

You can fool *some of* the people *all of* the time

There is a nonempty group of people so easily fooled that you can fool that group every time*

≡ There's (at least) one person you can fool every time

$\exists x\ \forall t\ \ person(x) \wedge time(t) \rightarrow canFool(x, t)$

For any given time, there is a non-empty group at that time that you can fool

≡ For every time, there's a person at that time that you can fool

$\forall t\ \exists x\ \ person(x) \wedge time(t) \rightarrow canFool(x, t)$

* Most common interpretation, I think

Translating English to FOL

You can fool *all of* the people *some of* the time

There's at least one time when you can fool everyone*

$\exists t\ \forall x\ time(t) \land person(x) \rightarrow canFool(x, t)$

Everybody can be fooled at some point in time

$\forall x\ \exists t\ person(x) \land time(t) \rightarrow canFool(x, t)$

* Most common interpretation, I think

Representation Design

- Many options for representing even a simple fact, e.g., something's color as red, green or blue, e.g.:
  - green(kermit)
  - color(kermit, green)
  - hasProperty(kermit, color, green)
- Choice can influence how easy it is to use
- Last option of representing properties & relations as triples used by modern knowledge graphs
  - Easy to ask: What color is Kermit? What are Kermit's properties?, What green things are there? Tell me everything you know, …

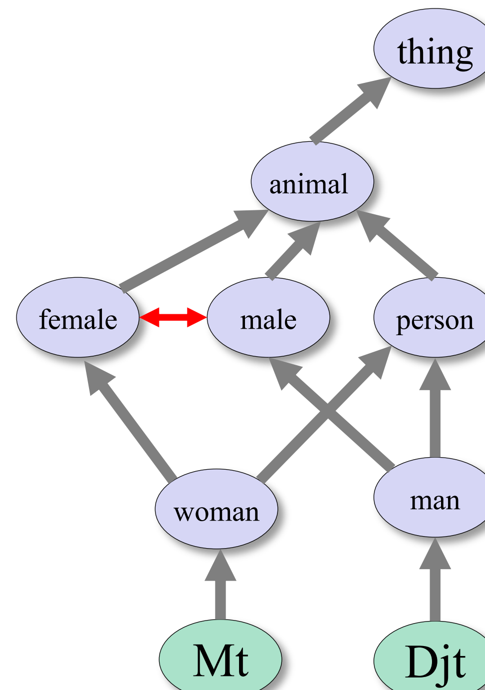
Simple genealogy KB in FOL

Design a knowledge base using FOL that

- Has facts of immediate family relations, e.g., spouses, parents, etc.
- Defines more complex relations (ancestors, relatives)
- Detect conflicts, e.g., you are your own parent
- Infers relations, e.g., grandparent from parent
- Answers queries about relationships between people

How do we approach this?

- Design an initial ontology of types, e.g.
  - person, animal, man, woman, …
- Types form a taxonomy or lattice*, e.g.
  - person(X) <=> man(X) ∨ woman(Y)
  - man(X) <=> person(X) ∧ male(X)
  - woman(X) <=> person(X) ∧ female(X)
  - female(X) <=> ~ male(X)
- Make assertions about individuals, e.g.
  - man(Djt)
  - woman(Mt)



* In a lattice, objects can have multiple immediate types

Extend with relations and constraints

- Simple two argument relations, e.g.
  - spouse, has_child, has_parent
- Add general constraints to relations, e.g.
  - spouse(X,Y) => ~ (X = Y)
  - spouse(X,Y) => person(X) ∧ person(Y)
  - spouse(X,Y) => (man(X) ∧ woman(Y)) ∨ (woman(X) ∧ man(Y))*
- Add FOL sentences for inference, e.g.
  - spouse(X,Y) ⇔ spouse(Y,X)
- Add instance data
  - e.g., spouse(Djt, Mt)

* Note this constraint is a traditional one than no longer holds

Example: A simple genealogy KB in FOL

Predicates:

- parent(x, y), child(x, y), father(x, y), daughter(x, y), etc.
- spouse(x, y), husband(x, y), wife(x,y)
- ancestor(x, y), descendant(x, y)
- male(x), female(y)
- relative(x, y)

Facts:

- husband(Joe, Mary), son(Fred, Joe)
- spouse(John, Nancy), male(John), son(Mark, Nancy)
- father(Jack, Nancy), daughter(Linda, Jack)
- daughter(Liz, Linda)
- etc.

Example Axioms

$(\forall x,y)\ parent(x, y) \leftrightarrow child(y, x)$

$(\forall x,y)\ father(x, y) \leftrightarrow parent(x, y) \wedge male(x)$

$(\forall x,y)\ mother(x, y) \leftrightarrow parent(x, y) \wedge female(x)$

$(\forall x,y)\ daughter(x, y) \leftrightarrow child(x, y) \wedge female(x)$

$(\forall x,y)\ son(x, y) \leftrightarrow child(x, y) \wedge male(x)$

$(\forall x,y)\ husband(x, y) \leftrightarrow spouse(x, y) \wedge male(x)$

$(\forall x,y)\ spouse(x, y) \leftrightarrow spouse(y, x)$

…

Axioms, definitions and theorems

- **Axioms**: facts and rules that capture (important) facts & concepts in a domain; used to prove **theorems**
  - Mathematicians dislike unnecessary (dependent) axioms, i.e., ones that can be derived from others
  - Dependent axioms can make reasoning faster, however
  - Choosing a good set of axioms is a design problem
- A **definition** of a predicate is of the form "p(X) $\leftrightarrow$ …" and can be decomposed into two parts
  - **Necessary** description: "p(x) $\rightarrow$ …"
  - **Sufficient** description "p(x) $\leftarrow$ …"
  - Some concepts have definitions (e.g., triangle) and some don't (e.g., person)

More on definitions

Example: define father(x, y) by parent(x, y) & male(x)

- **parent(x, y)** is a necessary (but not sufficient) description of father(x, y)

  father(x, y) $\rightarrow$ parent(x, y)

- **parent(x, y) ^ male(x) ^ age(x, 35)** is a sufficient (but not necessary) description of father(x, y):

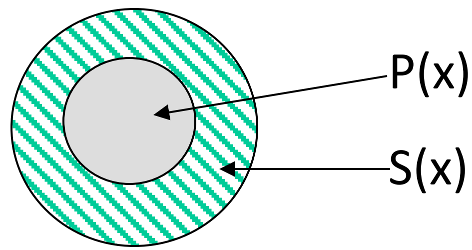
  father(x, y) $\leftarrow$ parent(x, y) ^ male(x) ^ age(x, 35)

- **parent(x, y) ^ male(x)** is a necessary and sufficient description of father(x, y)

  parent(x, y) ^ male(x) $\leftrightarrow$ father(x, y)

Another way to look at necessary and sufficient

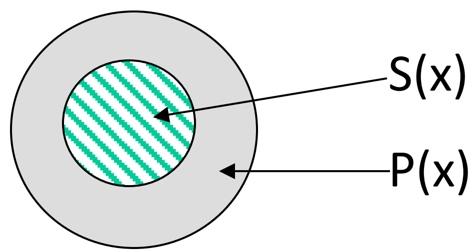
S(x) is a necessary condition of P(x)



 # all Ps are Ss

$(\forall x)$ P(x) => S(x)

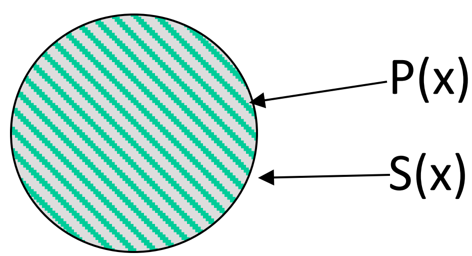
S(x) is a sufficient condition of P(x)



 # all Ps are Ss

$(\forall x)$ P(x) <= S(x)

S(x) is a necessary and sufficient condition of P(x)



 # all Ps are Ss

all Ss are Ps

$(\forall x)$ P(x) <=> S(x)

Higher-order logic

- FOL only lets us quantify over variables, and **variables can only range over objects**
- HOL allows us to quantify over relations, e.g.

  "two functions are equal iff they produce the same value for all arguments"

  $\forall f\ \forall g\ (f = g) \leftrightarrow (\forall x\ f(x) = g(x))$
- E.g.: (quantify over predicates)

  $\forall r\ transitive(\ r\ ) \rightarrow (\forall xyz)\ r(x,y) \wedge r(y,z) \rightarrow r(x,z))$
- More expressive, but reasoning is undecide-able, in general

Examples of FOL in use

- Semantics of W3C’s Semantic Web stack (RDF, RDFS, OWL) is defined in FOL
- OWL Full is equivalent to FOL
- Other OWL profiles support a subset of FOL and are more efficient
- FOL oriented knowledge representation systems have many user friendly tools
- E.g.: Protégé for creating, editing and exploring OWL ontologies

Examples of FOL in use

Many practical approaches embrace the approach that "some data is better than none"

- The semantics of schema.org is only defined in natural language text
- Wikidata's knowledge graph has a rich schema
  - Many constraint/logical violations are flagged with warnings
  - However, not all, see this Wikidata query that finds people who are their own mother or father

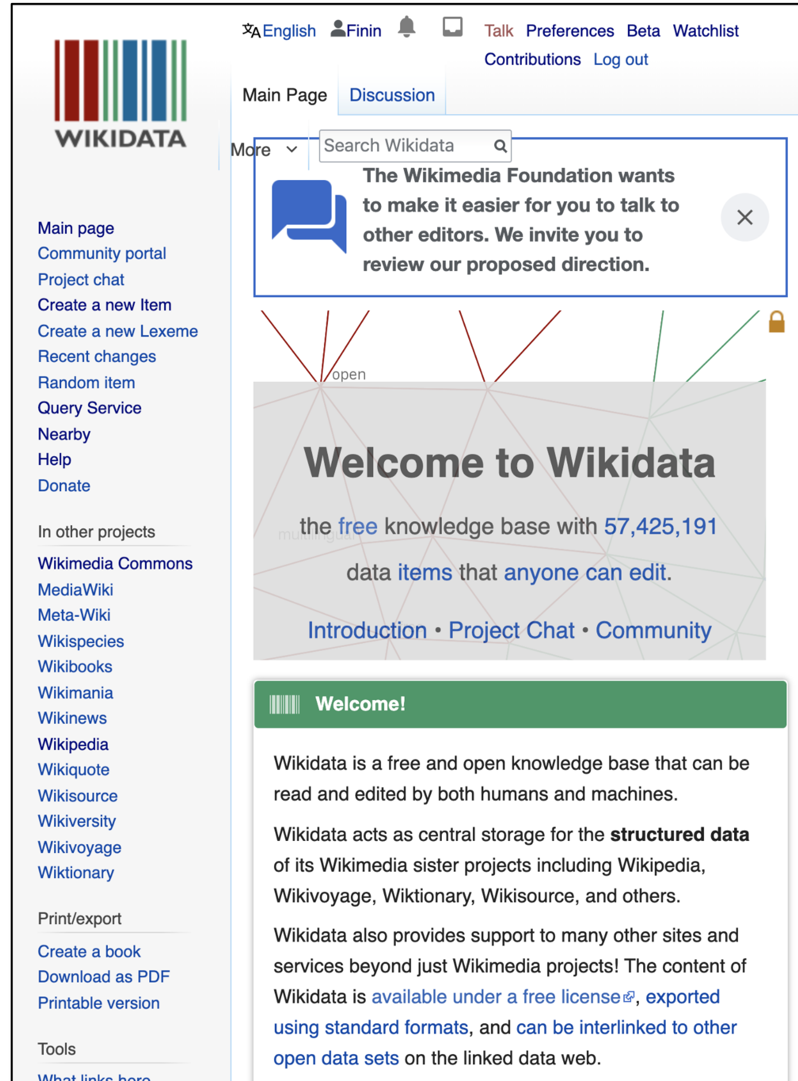
Wikidata

"collaboratively edited structured dataset used by Wikimedia sister projects and others"

Goal: consolidate *knowable facts* for use in the 200+ Wikipedia sites and other Wikimedia resources (e.g., in Infoboxes)

Integrates Wikimedia sites & links entities to ~7K external identifier systems (e.g., GeoNames ID)

Used by Google's Knowledge Graph, digital assistants (Siri, Alexa), Wikipedia Infoboxes, etc.



Wikidata

Knowledge graph with ~1B statements about ~93M items

Fine-grained **ontology** has ~2M types and ~5K properties

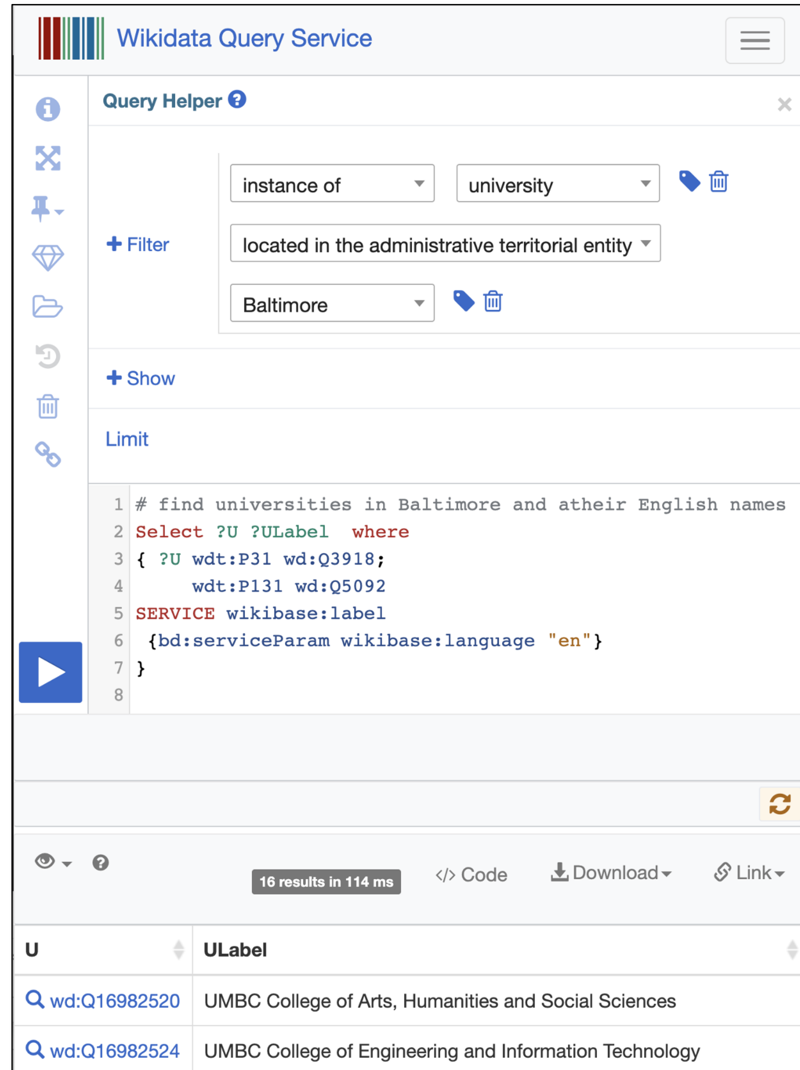
Data exposed as **RDF** triples

String values tagged with **language id**

Query using a standard **SPARQL** query service

Many community **tools** for search, visualization, update

Semantic Web realized !



Wikidata Entity

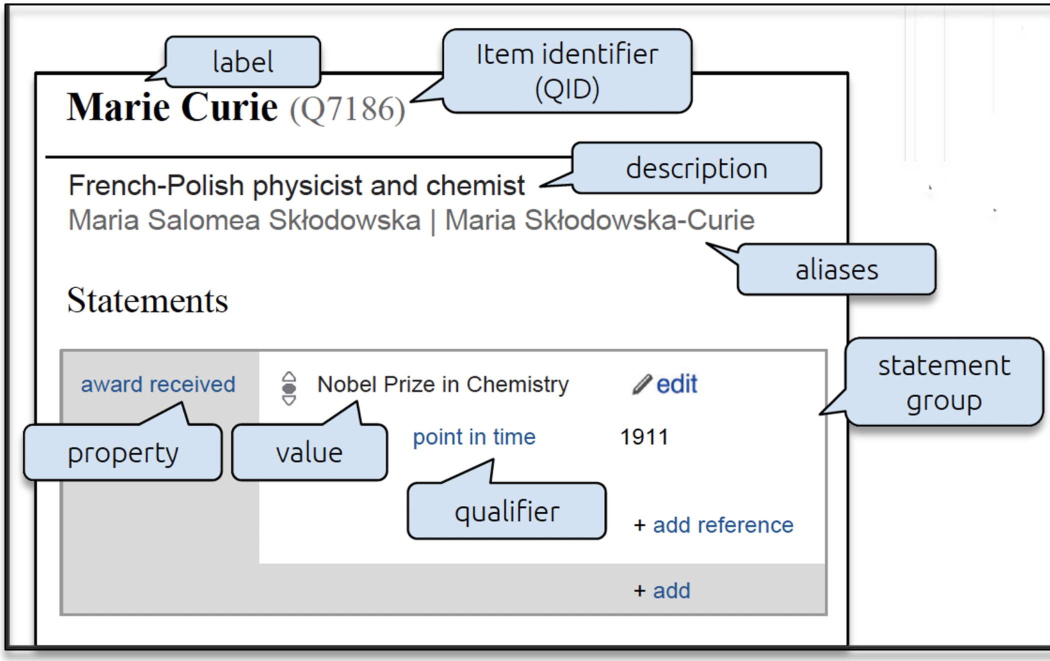
Each Wikipedia entity has

Unique ID, e.g. Q7186

A **label** (canonical name), short **description**, ≥0 **aliases** in a set of languages

One or more **types** (e.g., Q5)

Collection of **statements** with optional qualifiers, references



Marie Currie is a human:

[wd:Q7186 wdt:P31 wd:Q5]

Ontology exploration

How can we understand an ontology with more than two million types?

wdtaxonomy is a useful tool for exploring the ontology

Given a type (e.g, Q3918, university) you can quickly see

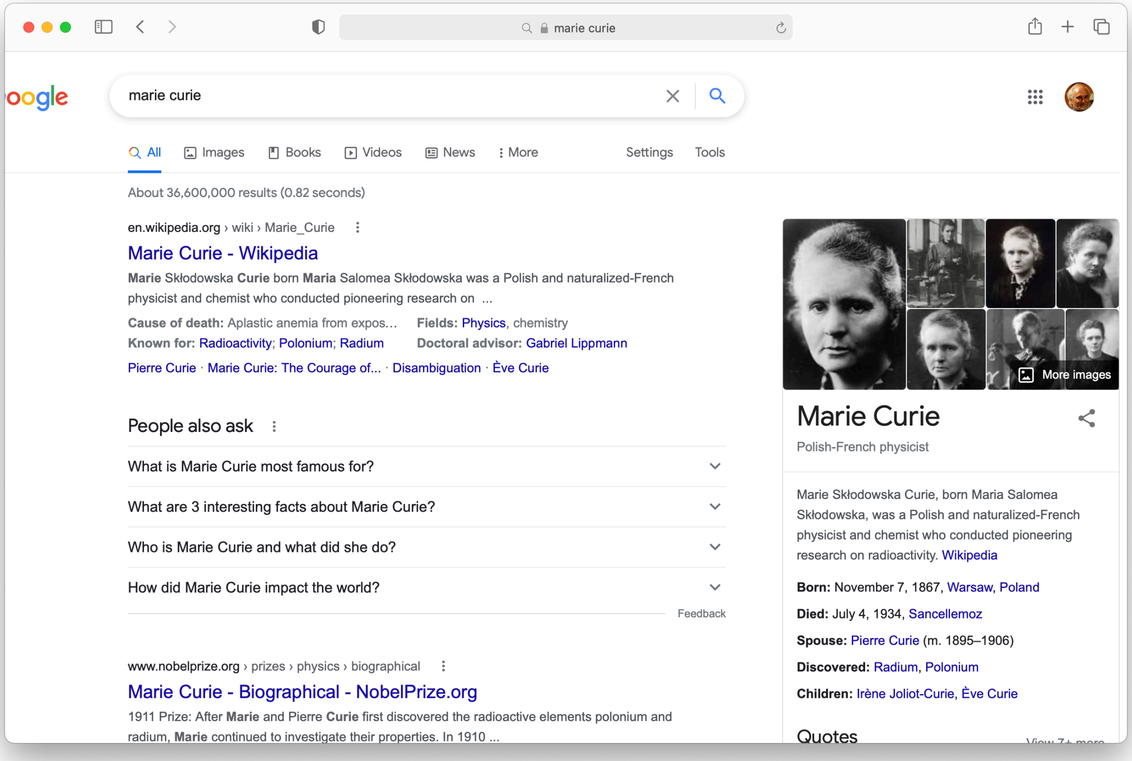
- Subtypes or supertypes (immediate or inferred),
- Number of instances (immediate or inferred),
- Direct instances
- Number Wikimedia sites it’s in

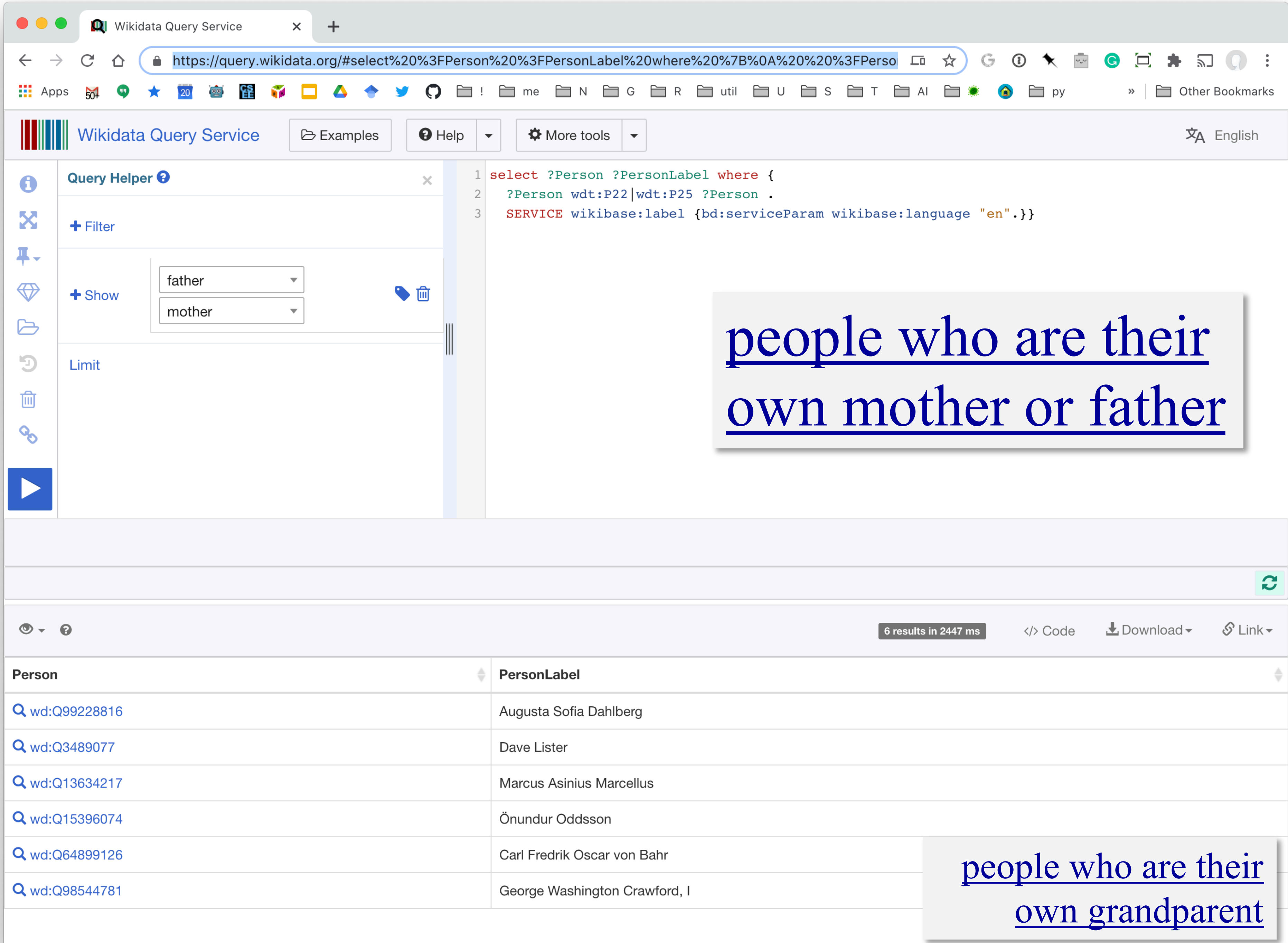
Implemented in javascript with a command line script

```
$$ wdtaxonomy Q3918 -c -t
university (Q3918) •163 ×15380 ↑↑
├──Universities in Germany (Q212462) •2
├──national university (Q265662) •11 ×73
├──National University (Q366354) •5
├──Imperial universities of Japan (Q562092) •12
├──Byzantine university (Q622870) •4
├──college and university rankings (Q847843) •23 ×45 ↑
├──public university (Q875538) •39 ×974 ↑
├──private university (Q902104) •32 ×846 ↑
├──new university (Q987075) •4 ×1
├──Red brick university (Q1202123) •11
├──??? (Q1305046) •2
├──institute of technology (Q1371037) •20 ×325
├──veterinary medicine school (Q1384955) •5 ×28
├──online university (Q1407393) •4 ×10 ↑
├──virtual university (Q1755248) •8 ×11
│ ╘══online university (Q1407393) •4 ×10 ↑ …
├──comprehensive university (Q1767829) •2 ×6
├──plate glass university (Q1902446) •8
├──medical university (Q1916585) •1 ×9 ↑
├──??? (Q2073922) •1
├──pontifical university (Q2120466) •18 ×37 ↑↑
├──Corporate university (Q2278672) •6
├──ancient university (Q2667285) •9 ×1
├──central university (Q3351682) •12 ×2
├──collegiate university (Q3354859) •9 ×12
├──deemed university (Q3520135) •6 ×16
├──university in France (Q3551775) •3 ×75 ↑
├──Istituto superiore per le industrie artistiche (Q3803831) •2 ×4
├──??? (Q3803846) •1 ×2
├──Smolny Institute for Noble Maidens (Q4432880) •1
├──??? (Q4475845) •2
├──federal university (Q4481793) •3 ×3
├──ecclesiastical university (Q5332280) •6 ×2
├──labor universities (Q5690751) •1 ×6
├──open university (Q6755402) •4 ×1
├──Urban university (Q7900184) •2
├──??? (Q10387922) •1
├──international university (Q10829188) •3 ×9
├──autonomous university (Q11057861) •2 ×1
├──research university (Q15936437) •9 ×224
├──Italian universities (Q20009854) •2
├──??? (Q20052016) •1 ×2
├──Canciller de Universidad (Q21547263)
├──imperial university of the Russian Empire (Q28667313) •2 ×12
├──universities in China (Q28700403) •1
├──Institute of National Importance (Q47531586) ×1 ↑
├──campusuniversity (Q59537665) ×3
├──Indiana University Bloomington Department of French and Italian (Q63441027)
├──Indiana University Department of French and Italian (Q63441251)
└──Indiana University Bloomington Department of History (Q63441447)
```

Wikidata and Infoboxes

- Web search engines use custom **knowledge graphs** for infoboxes with information about queried items
- Wikidata is an open-source knowledge graph that shares roots with these
- They all draw on the same knowledge, like the ~300 Wikipedia & Wikimedia sites
- Such knowledge graphs are often used by language understanding systems, e.g., for entity linking (who is [Michael Jordan](#))





people who are their own mother or father

people who are their own grandparent

FOL Summary

- First order logic (FOL) introduces predicates, functions and quantifiers
- More expressive, but reasoning more complex
  - Reasoning in propositional logic is NP hard, FOL is semi-decidable
- Common AI knowledge representation language
  - Other KR languages (e.g., OWL) are often defined by mapping them to FOL
- FOL variables range over objects
  - HOL variables range over functions, predicates or sentences
- Some practical systems avoid enforcing rigid FOL constraints due to having noisy data

Fín