A Glimpse of Game Theory

08

Games and Game Theory

- Much effort to develop computer programs for artificial games like chess or poker commonly played for entertainment
- Larger issue: account for, model, and predict how agents (human or artificial) interact with other agents
- **Game theory** accounts for mixture of cooperative and competitive behavior
- Applies to zero-sum and non-zero-sum games

Basic Ideas of Game Theory

- Game theory studies how strategic interactions among **rational players** produce **outcomes** with respect to players' **preferences**
  - Preferences represented as utilities (numbers)
  - Outcomes might not have been intended
- Provides a general theory of strategic behavior
- Generally depicted in mathematical form
- Plays important role in economics, decision theory and **multi-agent systems**

Zero Sum Games

- Zero-sum: participant's gain/loss exactly balanced by losses/gains of the other participants
- Total gains of participants minus total losses = 0

  Poker is zero sum game: money won = money lost
- Commercial trade not a zero-sum game

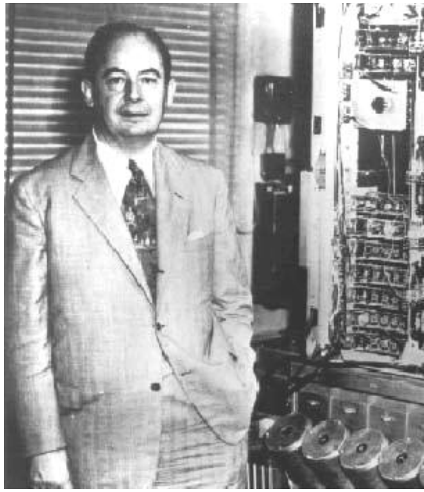
  If country with an excess of bananas trades with another for their excess of apples, both may benefit
- Non-zero-sum games more complex to analyze
- More non-zero-sum games as world becomes more complex, specialized, and interdependent

Rules, Strategies, Payoffs & Equilibrium

Situations are treated as “games”:

- **Rules** of game: who can do what, and when they can do it
- Player's **strategy**: plan for actions in each possible situation in the game
- Player's **payoff**: amount that player wins or loses in particular situation in a game
- Player has a dominant strategy if her best strategy doesn’t depend on what others do

Game Theory Roots

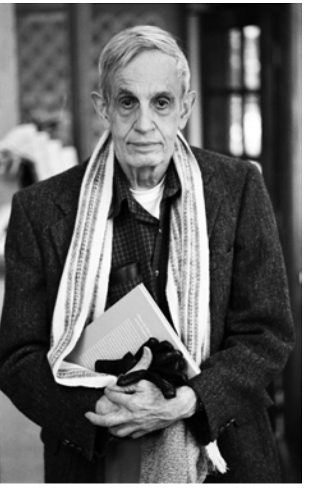


- Defined by John von Neumann & Oskar Morgenstern

  von Neumann, J., and Morgenstern, O., (1947).
  The Theory of Games and Economic Behavior.

- Provides powerful model & practical tools to model interactions among sets of autonomous agents
- Used to model strategic policies (e.g., arms race among countries)

Nash Equilibrium



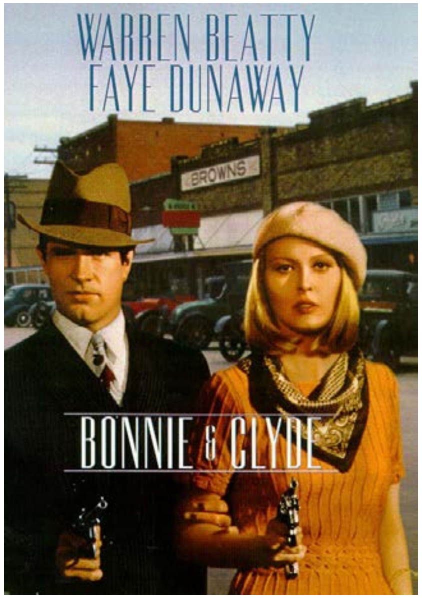
- Occurs when each player's strategy is optimal given strategies of other players
- It means that no player benefits by unilaterally changing strategy, while others stay fixed
- Like a local maximum in hill climbing
- Every finite game has at least one Nash equilibrium in either pure or mixed strategies (proved by John Nash)
  - J. F. Nash. 1950. Equilibrium Points in n-person Games. Proc. National Academy of Science, 36
  - Nash won 1994 Nobel Prize in economics for this work
  - Read A Beautiful Mind by Sylvia Nasar (1998) and/or see the 2001 film

Prisoner's Dilemma

Will the two prisoners cooperate to minimize total loss of liberty or will one of them, trusting the other to cooperate, betray him so as to go free?

- Famous example from game theory
- Strategies must be undertaken without full knowledge of what other players will do
- Players adopt dominant strategies, but they don't necessarily lead to the best outcome
- **Rational behavior** leads to a situation where **everyone is worse off**!

Bonnie and Clyde



Bonnie and Clyde are arrested and charged with crimes. They're questioned separately, unable to communicate. They know how it works:

- If both proclaim mutual innocence (cooperating), they will be found guilty anyway and get three-year sentences for robbery
- If one confesses (defecting) and the other doesn't (cooperating), the confessor is rewarded with a light, one-year sentence and the other gets a severe eight-year sentence
- If both confess (defecting), then the judge sentences both to a moderate four years in prison

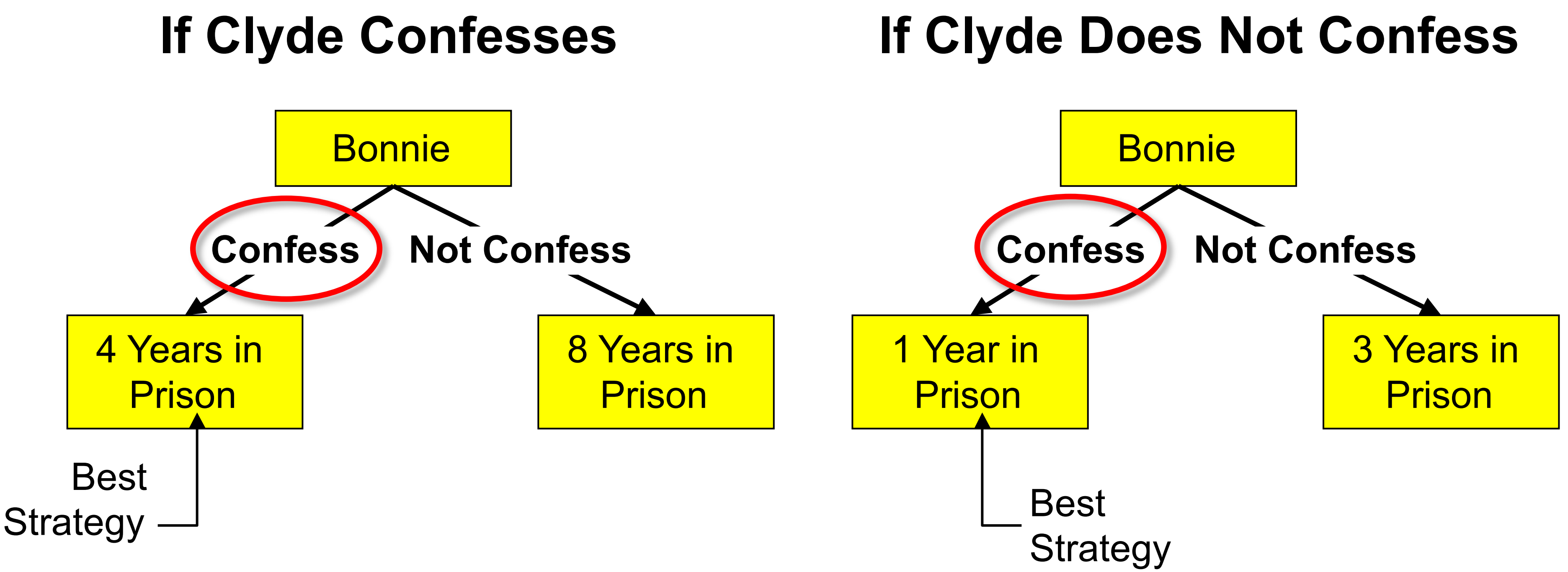
What should Bonnie do? What should Clyde do?

The payoff matrix

		CLYDE	
		Confess	**Not Confess**
BONNIE	**Confess**	4 years each	1 year for Bonnie and 8 years for Clyde
	Not Confess	8 years for Bonnie and 1 year for Clyde	3 years each

Bonnie’s Decision Tree

There are two cases to consider



Bonnie’s **Dominant strategy** is to confess (defect) because no matter what Clyde does, she is better off confessing

So what?

- Clyde's reasoning is the same
  - They both get 4-year sentences
  - They could have both had 3-years
- But it seems we should always defect and never cooperate
- No wonder Economics has been called the dismal science

Some PD examples

- There are lots of examples of the Prisoner's Dilemma situations in the real world
- It makes it difficult for "players" to avoid the bad outcome of both defecting
  - Cheating on a cartel
  - Trade wars between countries
  - Arms races between countries
  - Advertising
  - Communal coffee pot
  - Class team project

Cheating on a Cartel

Cartel: association of firms with purpose of maintaining prices at a high level and restricting competition

- Cartel members' possible strategies range from abiding by their agreement to cheating

  i.e., can charge the cartel price or a lower one
- Cheating firms can increase profits
- The best strategy is charging the low price

Trade Wars Between Countries

- Free trade benefits both trading countries
- Tariffs can benefit one trading country
- Imposing tariffs can be a dominant strategy and establish a Nash equilibrium even though it may be inefficient

Advertising

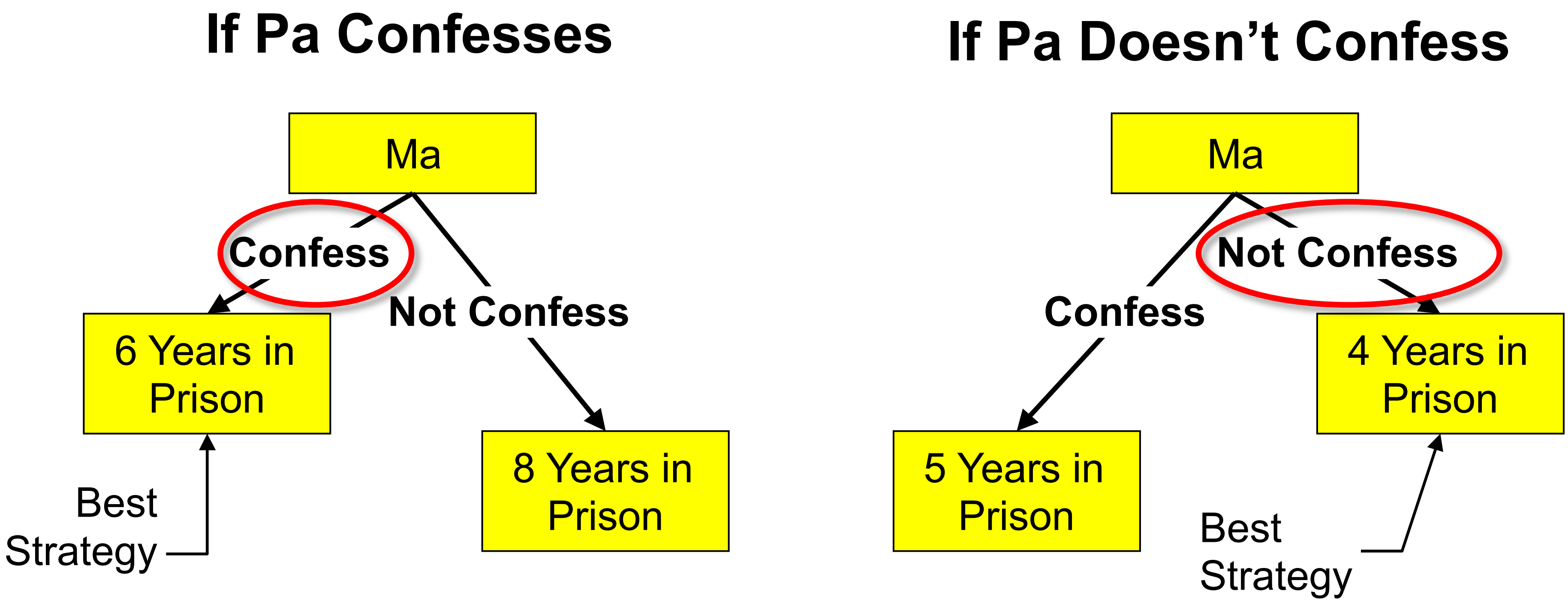
- Advertising is expensive
- All firms advertising tends to equalize the effects
- Everyone would gain if no one advertised
- But firms increase their advertising to gain advantage
- Which makes their competition do the same
- It's an arms race

Games Without Dominant Strategies

- In many games, players have no **dominant strategy**
- Player's strategy depends on others’ strategies
- If player's best strategy depends on another’s strategy, she has no dominant strategy

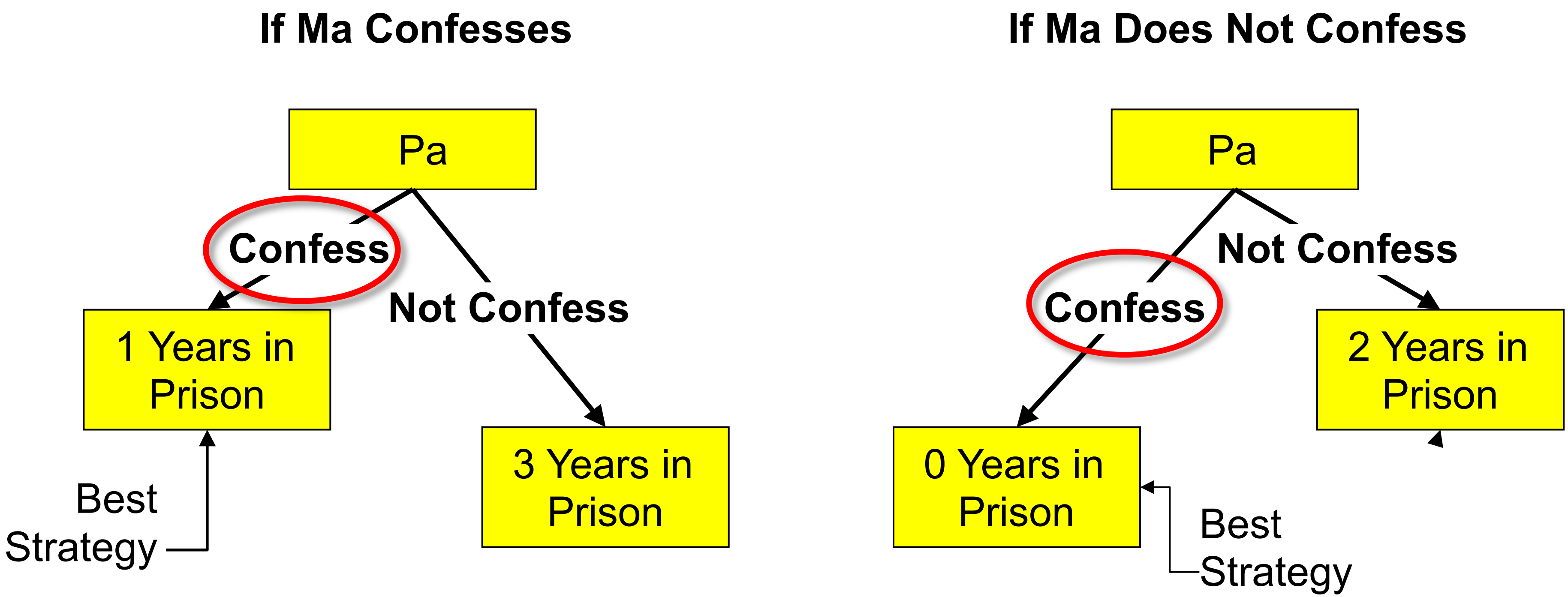
		Pa	
		Confess	**Not Confess**
Ma	**Confess**	6 years for Ma 1 year for Pa	5 years for Ma 3 years for Pa
	Not Confess	8 years for Ma 0 years for Pa	4 years for Ma 2 years for Pa

Ma’s Decision Tree



Ma has no explicit dominant strategy, but there is a best one since Pa does have a dominant strategy (What is it?)

Pa’s Decision Tree



Pa does have a dominant strategy: confess

So Ma’s best strategy is to confess

Some games have no simple solution

Neither player has a dominant strategy.
There is no non-cooperative solution

		Player B: 1	Player B: 2
Player A	1	1, -1	-1, 1
	2	-1, 1	1, -1

Best strategy for each is to randomly choose 1 or 2

Repeated Games

- A repeated game is a game that the same players play more than once
- Repeated games differ from one-shot games since a player’s current actions can depend on the past behavior of other players
- **Cooperation is encouraged**

Payoff matrix for the generic two person dilemma game

(A's payoff,
B's payoff)

where C: cooperate
And D: defect

Player B

Player A

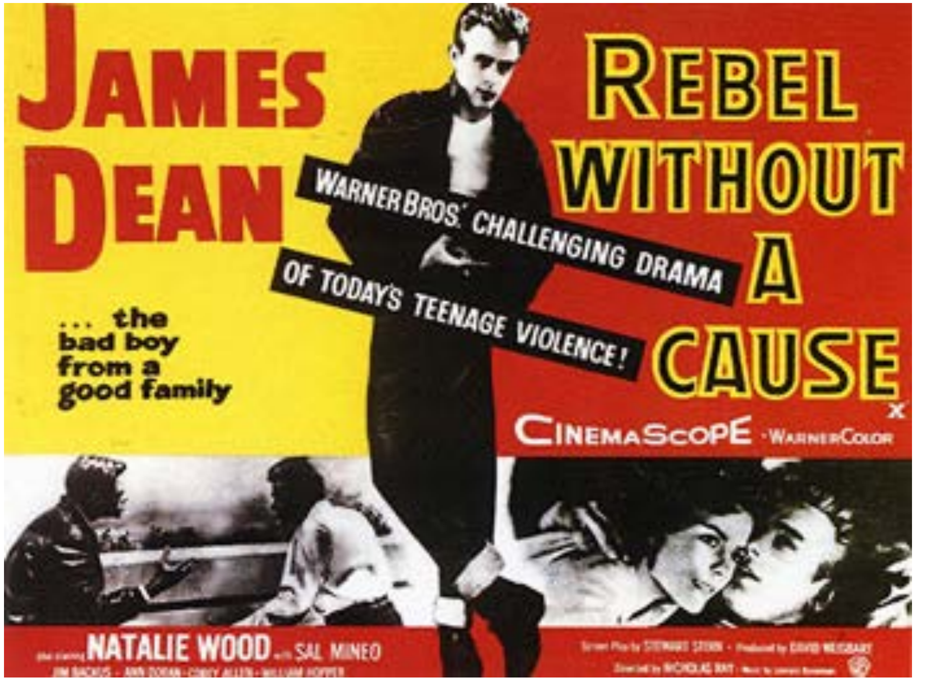
		cooperate	defect
Player A	cooperate	(CC,CC) reward for mutual cooperation	(CD,DC) sucker's payoff and temptation to defect
	defect	(DC,CD) temptation to defect and sucker's payoff	(DD,DD) punishment for mutual defection

Payoffs

- Four payoffs are involved
  - CC: Both players cooperate
  - CD: You cooperate, other defects (sucker's payoff)
  - DC: You defect, other cooperates (temptation to defect)
  - DD: Both players defect
- Assigning values induces an ordering, with 24 possibilities (4!); three lead to "dilemma" games
  - **Prisoner's dilemma:** DC > CC > DD > CD
  - **Chicken:** DC > CC > CD > DD
  - **Stag Hunt:** CC > DC > DD > CD

Chicken

- DC > CC > CD > DD
- Rebel without a cause scenario
- Two cars race toward one another
- Drivers choose to serve or not
  - Cooperation: swerving
  - Defecting: not swerving
- Optimal move: do exactly the opposite of other player



Stag Hunt

- CC > DC > DD > CD
- Two players on a stag hunt
- Hard task requiring coordination but with big shared payoff
- Hare seen, do you defect and chase it?
  - Cooperate: keep after the stag
  - Defect: switch to chasing hare
- Optimal play: do exactly what the other player(s) do

Prisoner's dilemma

- DC > CC > DD > CD
- Optimal play: always defect
- Two rational players will always defect.
- Thus, (naïve) individual rationality subverts their common good

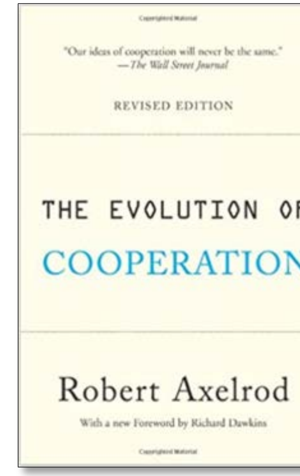
More examples of the PD in real life

- Communal coffeepot
  - Cooperate by making new pot of coffee if you take last cup
  - Defect by taking last cup and not making new pot, depending on the next coffee seeker to do it
  - DC > CC > DD > CD
- Class team project
  - Cooperate by doing your part well and on time
  - Defect by slacking, hoping other team members will come through and sharing benefits of good grade
  - (Arguable) DC > CC > DD > CD

Iterated Prisoner’s Dilemma

- Game theory: rational players should always defect when engaged in a PD situation
- In real situations, people don’t always do this
- Why not? Possible explanations:
  - People aren’t rational
  - Morality
  - Social pressure
  - Fear of consequences
  - Evolution of species-favoring genes
- Which make sense? How can we formalize?

Iterated Prisoner's Dilemma



- **Key idea:** We often play more than one "game" with a given player
- Players have knowledge of past games, including their choices and other players' choices
- Choice when playing against a player can be based on whether she's cooperated in past
- Simulation first done by Robert Axelrod where programs played in a round-robin tournament (DC=5;CC=3;DD=1;CD=0)
- The simplest program won!

Some possible strategies

- Always defect
- Always cooperate
- Randomly choose
- Pavlovian (win-stay, lose-switch)

  Start always cooperate, switch to always defect when punished by other's defection, switch back & forth on every punishment
- Tit-for-tat (TFT)

  Be nice, but punish defections: Start cooperating and, after that always do what other player did on previous round
- Joss

  Sneaky TFT that defects 10% of the time
- In an idealized (noise free) environment, TFT is both a very simple and very good strategy

Characteristics of Robust Strategies

Axelrod analyzed entries and identified characteristics

Nice: never defects first

Provocable: respond to defection by promptly defecting. Prompt response important; slow to anger a poor strategy; some programs tried even harder to take advantage

Forgiving: respond to single defections by defecting forever worked poorly. Better to respond to TIT with 0.9 TAT; might dampen echoes & prevent feuds

Clear: Clarity an important feature. With TFT you know what to expect and what will/won't work. With too much randomness or bizarre strategies in program, competing programs cannot analyze and began to always defect.

Implications of Robust Strategies

- Succeed not by "beating" others, but by allowing both to do well. TFT never "wins" a single turn! It can't. It can never do better than tie (all C).
- You do well by motivating cooperative behavior from others … the provocability part
- Envy is counterproductive. Doesn’t pay to get upset if someone does a few points better than you in a single encounter. To do well, others must also do well, e.g., business & its suppliers.

Implications of Robust Strategies

- Need not be smart to do well. TFT models cooperative relations with bacteria and hosts.
- Cosmic threats and promises aren’t necessary, though they may be helpful
- Central authority unnecessary, though it may be helpful
- Optimum strategy depends on environment. TFT not best program in all cases; too unforgiving of JOSS & too lenient with RANDOM

Emergence

- Process where larger entities, patterns, and regularities arise via interactions among smaller or simpler entities that themselves don’t exhibit such properties
- E.g.: Shape and behavior of a flock of birds or school of fish
- Might cooperation be an emergent property?

Required for emergent cooperation

- A **non-zero su**m situation
- **Players equal** in power; no discrimination or status differences
- **Repeated encounters** with others you can recognize

  Garages depending on repeat business versus those on busy highways. Being unlikely to ever see someone again => a non-iterated dilemma.
- **Low temptation payoff**

  If defecting makes you a billionaire, you're likely to do it. "Every person has a price"

Ecological model

- Assume ecological system supporting N players
- Players gain or loose points on each round
- After a round, worst players die, best multiply
- Environmental noise models that agent makes errors in following a strategy or misinterpret another's choice
- A simple way of modeling this is described in The Computational Beauty of Nature

Evolutionary stable strategies

- Strategies do better or worse against other strategies
- Successful strategies should work well in a variety of environments
  - E.g.: ALL-C works well in a mono-culture of ALL-Cs but not in a mixed environment
- Successful strategies can "fight off mutations"
  - E.g.: ALL-D mono-culture is very resistant to invasions by any cooperating strategies
  - E.g.: TFT can be "invaded" by ALL-C

Population simulation

(a) TFT wins

(b) A noise free version with TFT winning

(c) 0.5% noise lets Pavlov win

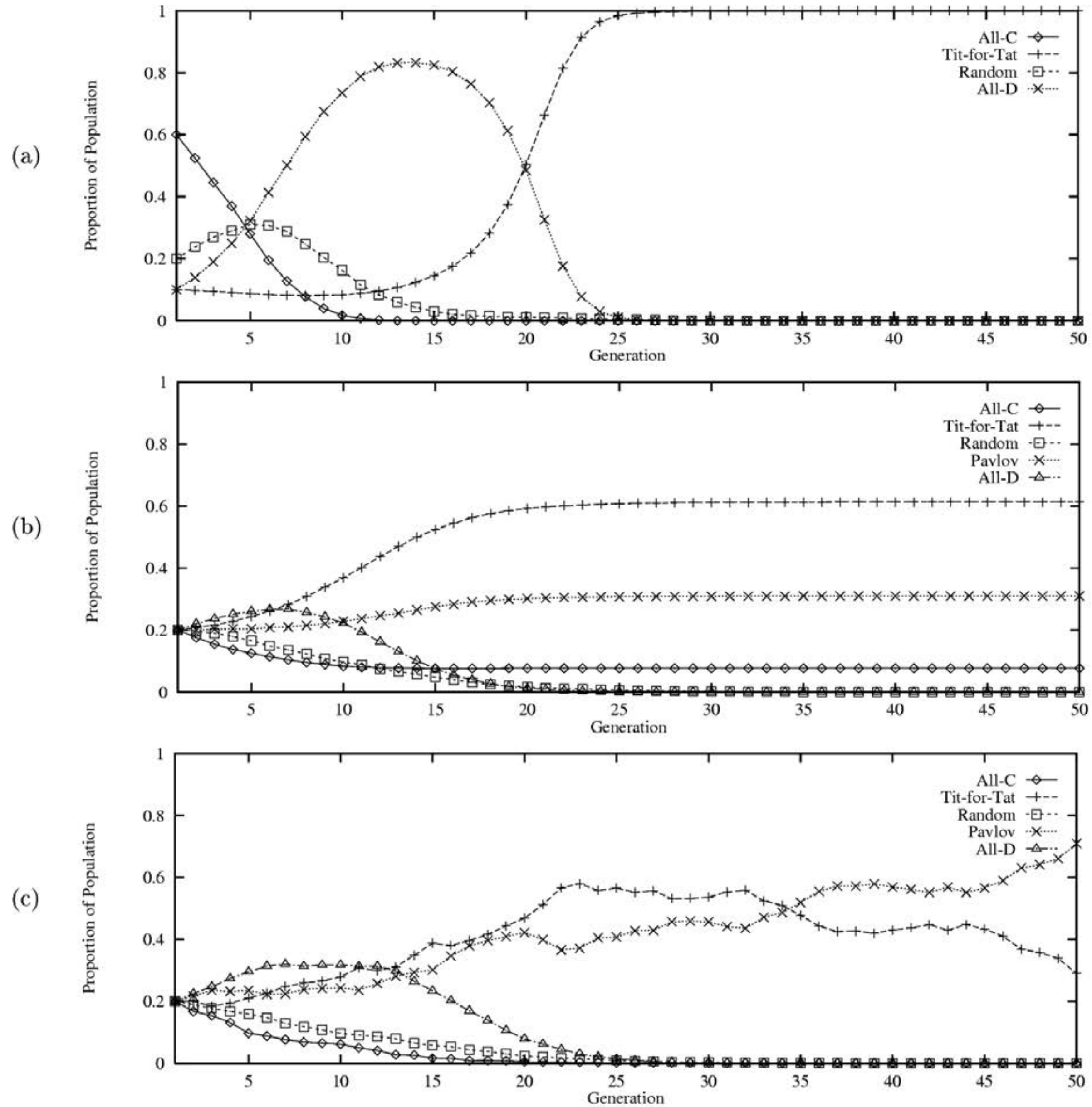


Figure 17.3 Population simulations of the ecological version of the iterated Prisoner's Dilemma: (a) an idealized version that illustrates the rise of **TFT**; (b) a noise-free simulation with **TFT** winning; (c) with 0.5 percent noise **PAV** wins

If you are interested…

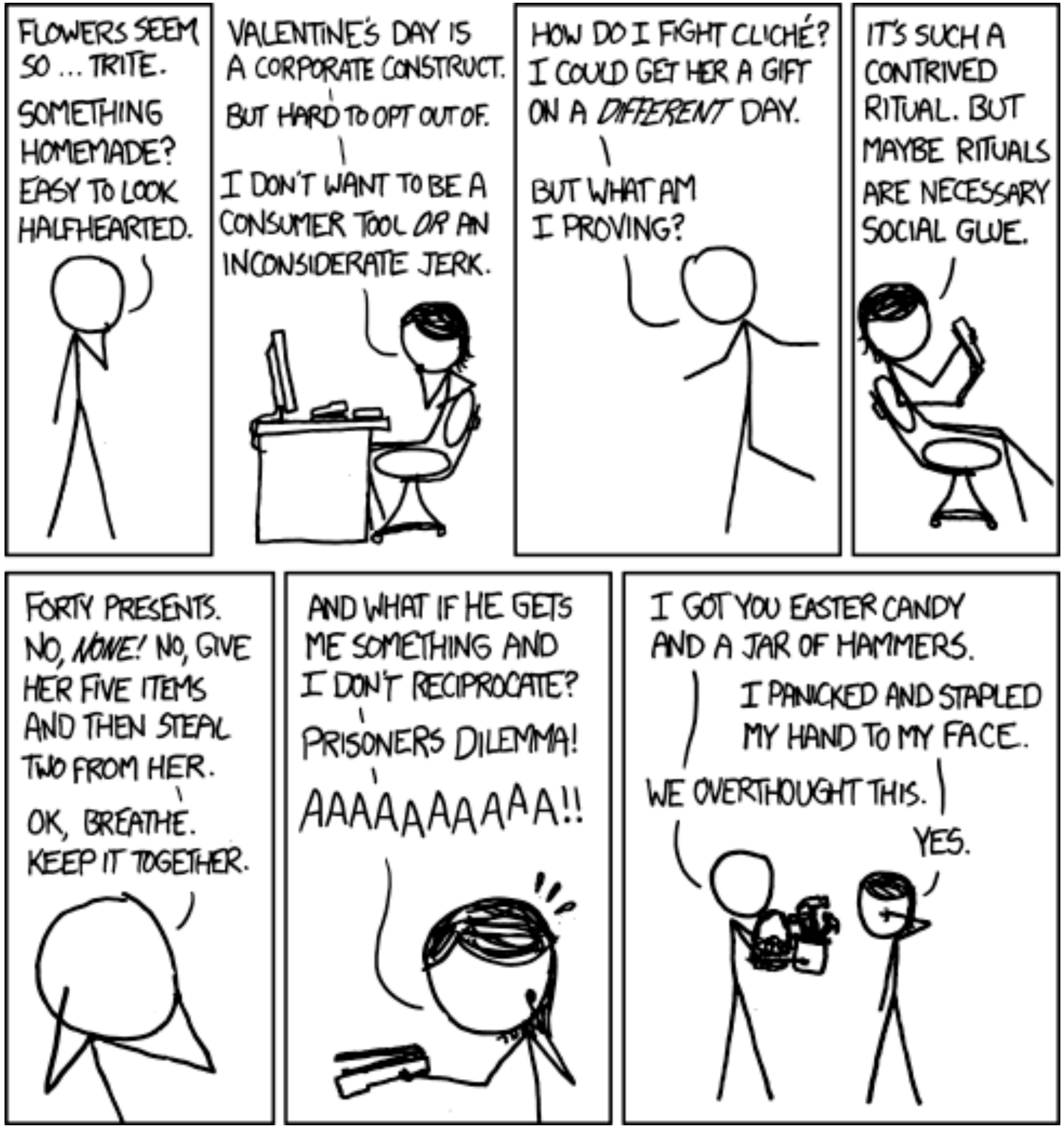
- [Axelrod Python]
  - https://github.com/Axelrod-Python
  - Explore strategies for the Prisoners dilemma game
  - Over 100 strategies from the literature and some original ones
  - Run round robin tournaments with a variety of options
  - Population dynamics
- Easy to install
  - pip install axelrod
- Also includes notebooks

20th anniversary IPD competition (2004)

- New Tack Wins Prisoner's Dilemma
- Coordinating Team Players within a Noisy Iterated Prisoner’s Dilemma Tournament
- U. Southhampton bot team won using covert channel to let Bots on the team recognize each other
- The 60 bots
  - Executed series of moves that signaled their ‘tribe’
  - Defect if other known to be outside tribe, coordin-ate if in tribe
  - Coordination was not just cooperation, but master/slave : defect/cooperate

Game Theory Relevance

- Game theory is important in more complex ”games”
  - E.g.: multiplayer, non-zero-sum, complicated payoffs
- Repeated games add complexity to balance cooperation and competition
- Used in multi-agent systems and where agents form teams with humans



The worst resolution to the Valentine Prisoner's Dilemma when YOU decide not to give your partner a present, but your PARTNER decides to testify against you in the armed robbery case.