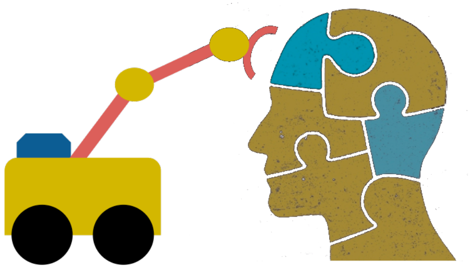


First-Order Logic (FOL) part 1

FOL Overview

- First Order logic (FOL) is a powerful knowledge representation (KR) system
- It’s used in AI systems in various ways, e.g.
  - To directly represent and reason about concepts and objects
  - To formally specify the meaning of other KR systems
  - To provide features that are useful in neural network deep learning systems

First-order logic

- First-order logic (FOL) models the world in terms of
  - **Objects,** which are things with individual identities
  - **Properties** of objects that distinguish them from others
  - **Relations** that hold among sets of objects
  - **Functions,** a subset of relations where there is only one "value" for any given "input"
- Examples:
  - Objects: students, lectures, companies, cars ...
  - Relations: brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
  - Properties: blue, oval, even, large, ...
  - Functions: father-of, best-friend, more-than ...

User provides

- **Constant symbols** representing individuals in world
  - BarackObama, Green, John, 3, “John Smith”
- **Predicate symbols,** map individuals to truth values
  - greater(5,3)
  - green(Grass)
  - color(Grass, Green)
  - hasBrother(John, Robert)
- **Function symbols,** map individuals to individuals
  - father_of(SashaObama) = BarackObama
  - color_of(Sky) = Blue

What do these mean?

- User should also indicate what these mean in a way that humans will understand
  - i.e., map to their own internal representations
- May be done via a combination of
  - Choosing good names for a formal terms, e.g. calling a concept HumanBeing instead of Q5
  - Comments in the definition `#human being`
  - Descriptions and examples in documentation
  - Reference to other representations , e.g., sameAs /m/0dgw95 in Freebase and Person in schema.org
  - Giving examples (Donald Trump) and non-examples (Luke Skywalker)

FOL Provides

- **Variable symbols**
  - E.g., x, y, foo
- **Connectives**
  - Same as propositional logic: not ($\neg$), and ($\wedge$), or ($\vee$), implies ($\rightarrow$), iff ($\leftrightarrow$)
- **Quantifiers**
  - Universal $\forall$**x** or **(Ax)**
  - Existential $\exists$**x** or **(Ex)**

Sentences: built from terms and atoms

- **term** (denoting an individual): constant or vari-able symbol, or n-place function of n terms, e.g.:
  - Constants: john, umbc
  - Variables: X, Y, Z
  - Functions: mother_of(john), phone(mother(x))
- **Ground terms** have no variables in them
  - **Ground:** john, father_of(father_of(john))
  - **Not Ground:** father_of(X)
- Syntax may vary: e.g., maybe variables must start with a “?” of a capital letter

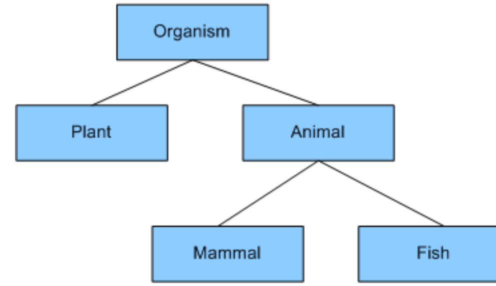
Sentences: built from terms and atoms

- **atomic sentences** (which are either true or false) are n-place predicates of n terms, e.g.:
  - green(kermit)
  - between(philadelphia, baltimore, dc)
  - loves(X, mother(X))
- **complex sentences** formed from atomic ones connected by the standard logical connectives with quantifiers if there are variables, e.g.:
  - loves(mary, john) $\vee$ loves(mary, bill)
  - $\forall$x loves(mary, x)

What do atomic sentences mean?

- Unary predicates typically encode a **type**
  - muppet(Kermit): kermit is a kind of muppet
  - green(kermit): kermit is a kind of green thing
  - integer(X): x is a kind of integer
- Non-unary predicates typically encode relations or properties
  - Loves(john, mary)
  - Greater_than(2, 1)
  - Between(newYork, philadelphia, baltimore)
  - hasName(john, “John Smith”)

Ontology



- Designing a logic representation is like designing a model in an object-oriented language
- **Ontology:** a "formal naming and definition of the types, properties and relations of entities for a domain of discourse"
- E.g.: schema.org ontology used to put semantic data on Web pages to help search engines
  - Here's the semantic markup Google sees on our 471 class site

Sentences: built from terms and atoms

- **quantified sentences** adds quantifiers ∀ and ∃
  - ∀x loves(x, mother(x))
  - ∃x number(x) ∧ greater(x, 100), prime(x)
- **well-formed formula (wff):** a sentence with no *free* variables or where all variables are *bound* by a universal or existential *quantifier*
  - In **(∀x)P(x, y)** x is bound & y is free so it's not a wff

Quantifiers: ∀ and ∃

- Universal **quantification**
  - (∀x)P(X) means P holds for **all** values of X in the domain associated with variable[1]
  - E.g., (∀X) dolphin(X) → mammal(X)
- Existential **quantification**
  - (∃x)P(X) means P holds for **some** value of X in domain associated with variable
  - E.g., (∃**X**) mammal(X) ∧ lays_eggs(X)
  - This lets us make statements about an object without identifying it

[1] a variable's domain is often not explicitly stated and is assumed by the context

Universal Quantifier: $\forall$

- Universal quantifiers typically used with *implies* to form *rules*:

  Logic: $(\forall X)\ student(X) \rightarrow smart(X)$

  Means: All students are smart

- Universal quantification *rarely* used without implies:

  Logic: $(\forall X)\ student(X) \wedge smart(X)$

  Means: Everything is a student and is smart

Existential Quantifier: $\exists$

- Existential quantifiers usually used with **and** to specify a list of properties about an individual

  Logic: $(\exists X)\ student(X) \wedge smart(X)$

  Meaning: There is a student who is smart

- Common mistake: represent this in FOL as:

  Logic: $(\exists X)\ student(X) \rightarrow smart(X)$

  Meaning: ?

Existential Quantifier: ∃

- Existential quantifiers usually used with **and** to specify a list of properties about an individual

  Logic: $(\exists X)\ student(X) \wedge smart(X)$

  Meaning: There is a student who is smart

- Common mistake: represent this in FOL as:

  Logic: $(\exists X)\ student(X) \rightarrow smart(X)$

  $P \rightarrow Q = {\sim}P \vee Q$

  $\exists X\ student(X) \rightarrow smart(X) = \exists X\ {\sim}student(X) \vee smart(X)$

  Meaning: There's something that is either not a student or is smart

Quantifier Scope

- FOL sentences have structure, like programs
- In particular, variables in a sentence have a **scope**
- Suppose we want to say “everyone who is alive loves someone”

  $(\forall X)\ alive(X) \rightarrow (\exists\ Y)\ loves(X, Y)$

- Here’s how we scope the variables

$$(\forall X)\ alive(X) \rightarrow (\exists Y)\ loves(X, Y)$$

Scope of x

Scope of y

Quantifier Scope

- **Switching order of universal quantifiers *does not* change the meaning**
  - $(\forall X)(\forall Y)P(X,Y) \leftrightarrow (\forall Y)(\forall X)\ P(X,Y)$
  - Dogs hate cats (i.e., all dogs hate all cats)
- **You can switch order of existential quantifiers**
  - $(\exists X)(\exists Y)P(X,Y) \leftrightarrow (\exists Y)(\exists X)\ P(X,Y)$
  - A cat killed a dog
- **Switching order of universal and existential quantifiers *does* change meaning:**
  - Everyone likes someone: $(\forall X)(\exists Y)$ likes(X,Y)
  - Someone is liked by everyone: $(\exists Y)(\forall X)$ likes(X,Y)

```
def verify1():
    # Everyone likes someone: (∀x)(∃y) likes(x,y)
    for p1 in people():
        foundLike = False
        for p2 in people():
            if likes(p1, p2):
                foundLike = True
                break
        if not foundLike:
            print(p1, 'does not like anyone ☹')
            return False
    return True
```

Every person has at least one individual that they like.

Procedural example 1

```
def verify2():
    # Someone is liked by everyone: (∃y)( ∀x) likes(x,y)
    for p2 in people():
        foundHater = False
        for p1 in people():
            if not likes(p1, p2):
                foundHater = True
                break
        if not foundHater
            print(p2, ‘is liked by everyone ☺’)
            return True
    return False
```

There is a person who is liked by every person in the universe.

Procedural example 2

Connections between ∀ and ∃

- We can relate sentences involving ∀ and ∃ using extensions to **De Morgan's laws**:
  1. $(\forall x)\ P(x) \leftrightarrow \neg(\exists x) \neg\ P(x)$
  2. $\neg(\forall x)\ P(x) \leftrightarrow (\exists x) \neg P(x)$
  3. $(\exists\ x)\ P(x) \leftrightarrow \neg\ (\forall\ x) \neg P(x)$
  4. $\neg(\exists x)\ P(x) \leftrightarrow (\forall x) \neg P(x)$
- Examples
  1. All dogs don't like cats ↔ No dog likes cats
  2. Not all dogs bark ↔ There is a dog that doesn't bark
  3. All dogs sleep ↔ There is no dog that doesn't sleep
  4. There is a dog that talks ↔ Not all dogs can't talk

Notational differences

- **Different symbols** for *and, or, not, implies, ...*
  - $\forall \; \exists \; \Rightarrow \; \Leftrightarrow \; \wedge \; \vee \; \neg \; \bullet \; \supset$
  - p v (q ^ r)
  - p + (q * r)
- **Prolog**

```
cat(X) :- furry(X), meows (X), has(X, claws)
```

- **Lispy notations**

```
(forall ?x (implies (and (furry ?x)
                         (meows ?x)
                         (has ?x claws))
                    (cat ?x)))
```

Fin