Propositional Logic: Pro & Con

Propositional logic: pro and con

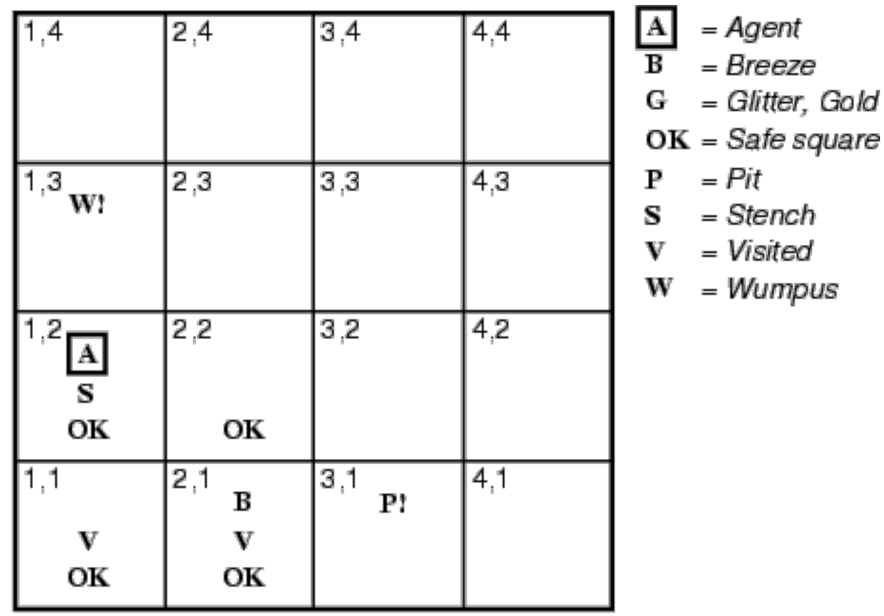
- **Advantages**
  - Simple KR language good for many problems
  - Lays foundation for higher logics (e.g., FOL)
  - Reasoning is decidable, though NP complete; efficient techniques exist for many problems
- **Disadvantages**
  - Not expressive enough for most problems
  - Even when it is, it can very “un-concise”

PL is a weak KR language

- Hard to identify *individuals* (e.g., Mary, 3)
- Can't directly represent properties of individuals or relations between them (e.g., "Bill age 24")
- Generalizations, patterns, regularities hard to represent (e.g., "all triangles have 3 sides")
- First-Order Logic (FOL) represents this informa-tion via **relations**, **variables** & **quantifier**s, e.g.,
  - *John loves Mary: loves(John, Mary)*
  - *Every elephant is gray:* $\forall$ x (elephant(x) → gray(x))
  - *There is a black swan:* $\exists$ x (swan(X) ^ black(X))

Hunt the Wumpus domain

- Some atomic propositions:
  - A12 = agent is in call (1,2)
  - S12 = There's a stench in cell (1,2)
  - B34 = There's a breeze in cell (3,4)
  - W22 = Wumpus is in cell (2,2)
  - V11 = We've visited cell (1,1)
  - OK11 = cell (1,1) is safe
  - …



- Some rules:
  - $\neg S22 \rightarrow \neg W12 \wedge \neg W23 \wedge \neg W32 \wedge \neg W21$
  - $S22 \rightarrow W12 \vee W23 \vee W32 \vee W21$
  - $B22 \rightarrow P12 \vee P23 \vee P32 \vee P21$
  - $W22 \rightarrow S12 \wedge S23 \wedge S32 \wedge W21$
  - $W22 \rightarrow \neg W11 \wedge \neg W21 \wedge \ldots \neg W44$
  - $A22 \rightarrow V22$
  - $A22 \rightarrow \neg W11 \wedge \neg W21 \wedge \ldots \neg W44$
  - $V22 \rightarrow OK22$

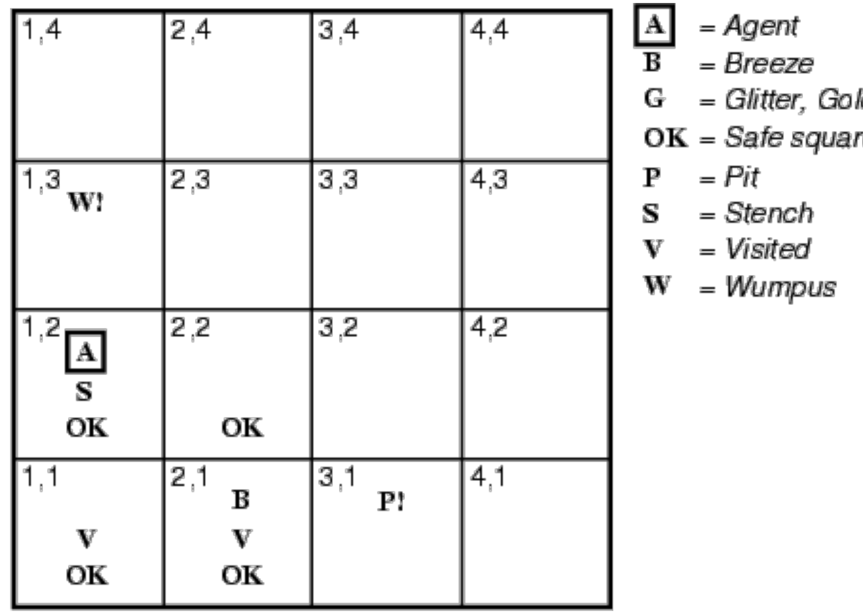
If there's no stench in cell 2,2 then the Wumpus isn't in cell 21, 23 32 or 21

Hunt the Wumpus domain

- Eight symbols for each cell, i.e.: A11, B11, G11, OK11, P11, S11, V11, W11
- Lack of variables requires giving similar rules for each cell!
- Ten rules (I think) for each

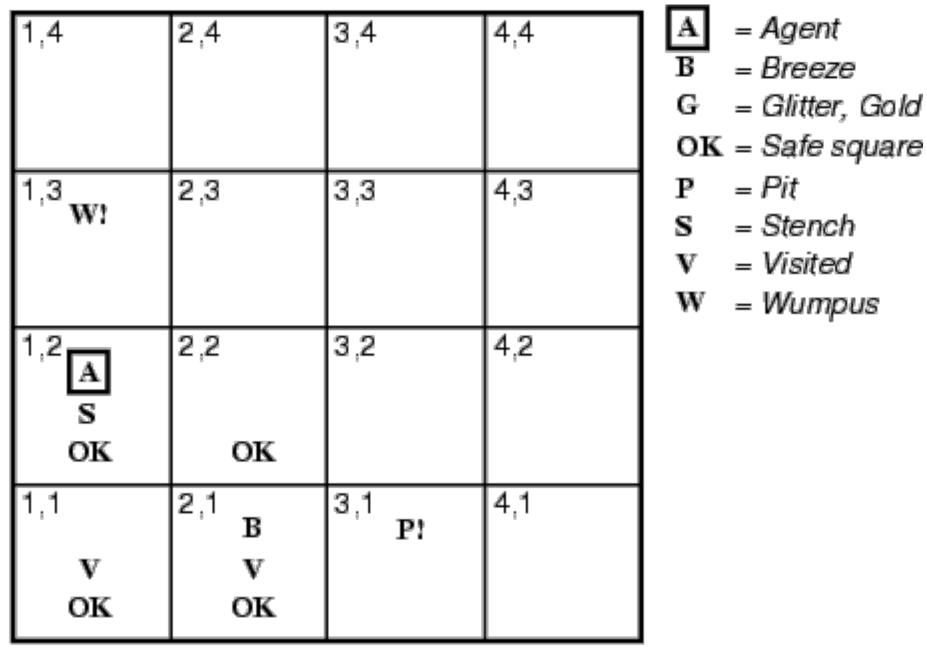
A11 → …
V11 → …
P11 → …
¬P11 → …

W11 → …
¬W11 → …
S11 → …
¬S11 → …
B11 → …
¬B11 → …



- 8 symbols for 16 cells => 128 symbols
- 2^{128} possible models ☹
- Must do better than brute force

After third move



- We can prove that the Wumpus is in (1,3) using these four rules
- See R&N section 7.5

(R1) $\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$

(R2) $\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$

(R3) $\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$

(R4) $S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$

Proving W13: Wumpus is in cell 1,3

(R1)	$\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$
(R2)	$\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$
(R3)	$\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$
(R4)	$S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$

Apply **MP** with $\neg$S11 and R1:

$\neg W11 \wedge \neg W12 \wedge \neg W21$

Apply **AE**, yielding three sentences:

$\neg W11, \neg W12, \neg W21$

Apply **MP** to ~S21 and R2, then apply **AE**:

$\neg W22, \neg W21, \neg W31$

Apply **MP** to S12 and R4 to obtain:

$W13 \vee W12 \vee W22 \vee W11$

Apply **UR** on ($W13 \vee W12 \vee W22 \vee W11$) and $\neg$W11:

$W13 \vee W12 \vee W22$

Apply **UR** with ($W13 \vee W12 \vee W22$) and $\neg$W22:

$W13 \vee W12$

Apply **UR** with ($W13 \vee W12$) and $\neg$W12:

W13

QED

Rule Abbreviation

MP: modes ponens

AE: and elimination

R: unit resolution

Propositional Wumpus problems

- Lack of variables prevents general rules, e.g.:
  - $\forall$ x, y V(x,y) → OK(x,y)
  - $\forall$ x, y S(x,y) → W(x-1,y) $\vee$ W(x+1,y) …
- Change of KB over time difficult to represent
  - In classical logic; a fact is true or false for all time
  - A standard technique is to index dynamic facts with the time when they're true
    - A(1, 1, 0) *# agent was in cell 1,1 at time 0*
    - A(2, 1, 1) *# agent was in cell 2,1 at time 1*
  - Thus we have a separate KB for every time point

Propositional logic summary

- **Inference**: process of deriving new sentences from old
  - **Sound** inference derives true conclusions given true premises
  - **Complete** inference derives all true conclusions from premises
- Different logics make different **commitments** about what the world is made of and the kind of beliefs we can have
- **Propositional logic** commits only to existence of facts that may or may not be the case in the world being represented
  - Simple syntax & semantics illustrates the process of inference
  - It can become impractical, even for very small worlds

Fin