Reasoning with Propositional Logic

Chapter 7.4–7.8

Some material adopted from notes by Andreas Geyer-Schulz and Chuck Dyer

Overview

- There are many ways to approach reasoning with propositional logic
- We'll look at one, resolution, that can be extended to first order logic
- Later will look approaches that are special to propositional logic.

Reasoning / Inference

- **Logical inference** creates new sentences that logically follow from a set of sentences (KB)
- It can also detect if a KB is inconsistent, i.e., has sentences that entail a **contradiction**
- An inference rule is **sound** if every sentence X it produces from a KB logically follows from the KB
  - i.e., inference rule creates no contradictions
- An inference rule is **complete** if it can produce every expression that logically follows from (is entailed by) the KB
  - Note analogy to complete search algorithms

Sound rules of inference

Examples of sound rules of inference

Each can be shown to be sound using a truth table

RULE	**PREMISE**	**CONCLUSION**
Modus Ponens	$A, A \rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Double Negation	$\neg\neg A$	A
Unit Resolution	$A \vee B, \neg B$	A
Resolution	$\mathbf{A \vee B, \neg B \vee C}$	$\mathbf{A \vee C}$

Resolution

- **Resolution** is a valid inference rule producing a new clause implied by two clauses containing *complementary literals*

  Literal: atomic symbol or its negation, i.e., P, ~P

- Amazingly, this is the **only** interference rule needed to build a sound & complete theorem prover
  - Based on proof by contradiction, usually called resolution refutation
- The resolution rule was discovered by Alan Robinson (CS, U. of Syracuse) in the mid 1960s

Resolution

- A KB is a set of sentences all of which are true, i.e., a conjunction of sentences
- To use resolution, put KB into *conjunctive normal form* (CNF)
  - Each sentence is a disjunction of one or more literals (positive or negative atoms)
- Every KB can be put into CNF, it's just a matter of rewriting its sentences using standard tautologies, e.g.:
  - $P \rightarrow Q \equiv \sim P \vee Q$

Resolution Example

Tautologies

$(A \rightarrow B) \leftrightarrow (\sim A \vee B)$

$(A \vee (B \wedge C)) \leftrightarrow (A \vee B) \wedge (A \vee C)$

- KB: $[P \rightarrow Q \,,\, Q \rightarrow R \wedge S]$
- KB: $[P \rightarrow Q \,,\, Q \rightarrow R,\ Q \rightarrow S\ ]$
- KB in CNF: $[\sim P \vee Q \,,\, \sim Q \vee R \,,\, \sim Q \vee S]$
- Resolve KB[0] and KB[1] producing:

  $\sim P \vee R$ *(i.e., $P \rightarrow R$)*
- Resolve KB[0] and KB[2] producing:

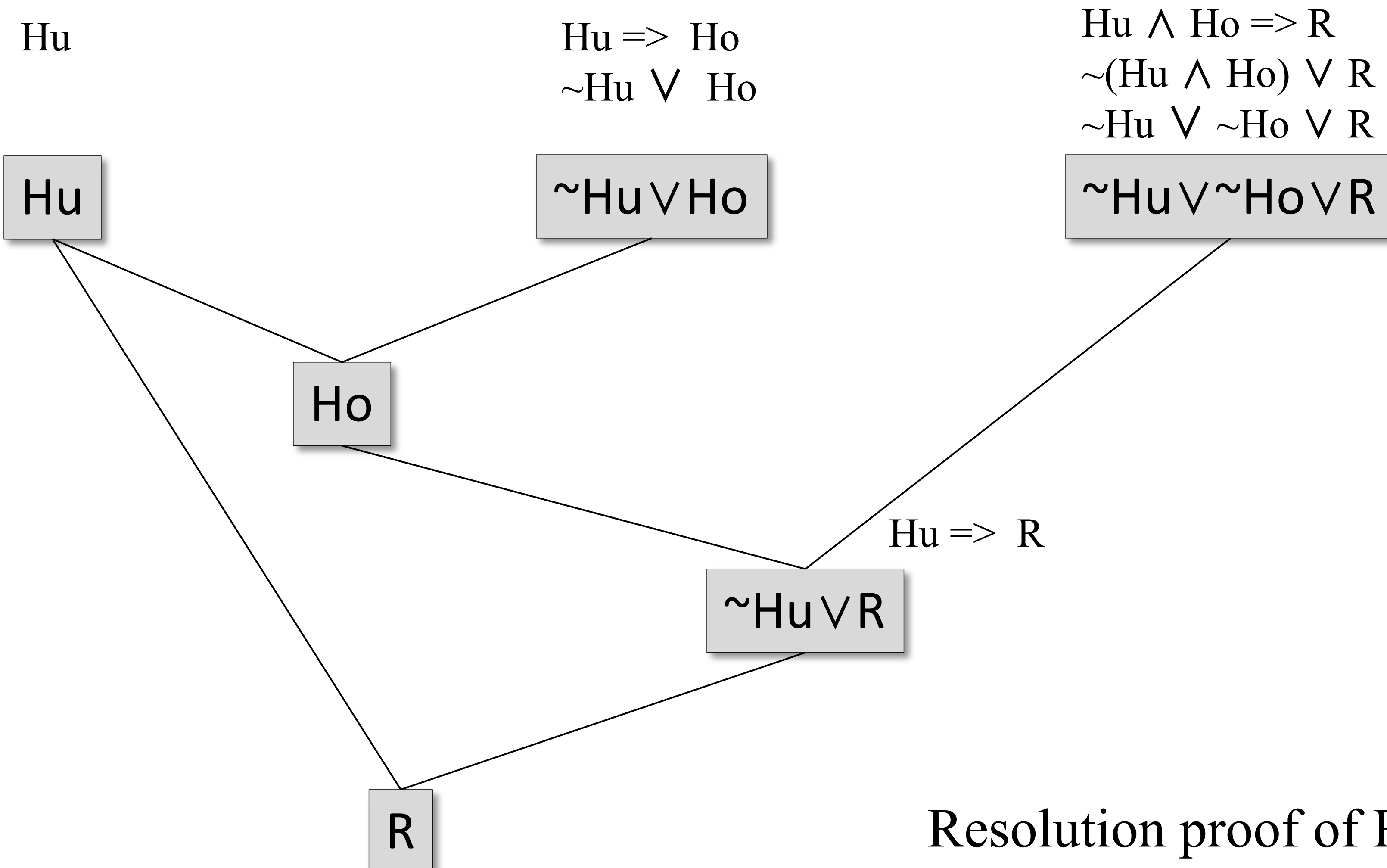
  $\sim P \vee S$ *(i.e., $P \rightarrow S$)*
- New KB: $[\sim P \vee Q \,,\, \sim Q \vee R,\ \sim Q \vee S,\ \sim P \vee R,\ \sim P \vee S]$

Proving it's raining with rules

- A **proof** is a sequence of sentences, where each is a premise (i.e., a given) or is derived from earlier sentences in the proof by an inference rule
- Last sentence is the **theorem** (also called goal or query) that we want to prove
- The *weather problem* using traditional reasoning

1 Hu	premise	"It's humid"
2 $Hu \rightarrow Ho$	premise	"If it's humid, it's hot"
3 Ho	modus ponens(1,2)	"It's hot"
4 $(Ho \wedge Hu) \rightarrow R$	premise	"If it's hot & humid, it's raining"
5 $Ho \wedge Hu$	and introduction(1,3)	"It's hot and humid"
6 R	modus ponens(4,5)	"It's raining"

Proving it's raining with resolution



Resolution proof of R

A simple proof procedure

This procedure generates new sentences in a KB

1. Convert all sentences in the KB to CNF[1]
2. Find all pairs of sentences in KB with complementary literals[2] that have not yet been resolved
3. If there are no pairs stop else resolve each pair, adding the result to the KB and go to 2

- Is it sound?
- Is it complete?
- Will it always terminate?

1: Conjunctive normal form is a conjunction of disjunctive sentences

2: a literal is a variable or its negation

Resolution refutation

1. Add negation of goal to the KB
2. Convert all sentences in KB to CNF
3. Find all pairs of sentences in KB with complementary literals that have not yet been resolved
4. If there are no pairs stop else resolve each pair, adding the result to the KB and go to 2

- If we derived an empty clause (i.e., a contradiction) then the conclusion follows from the KB
- If we did not, the conclusion cannot be proved from the KB

Fin