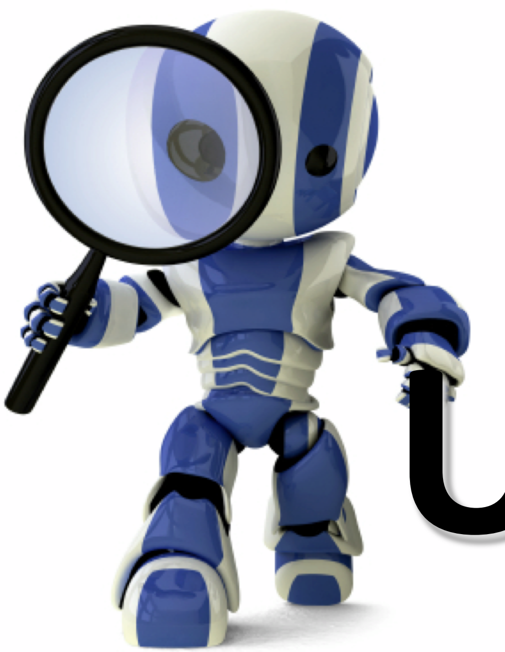


Uninformed Search

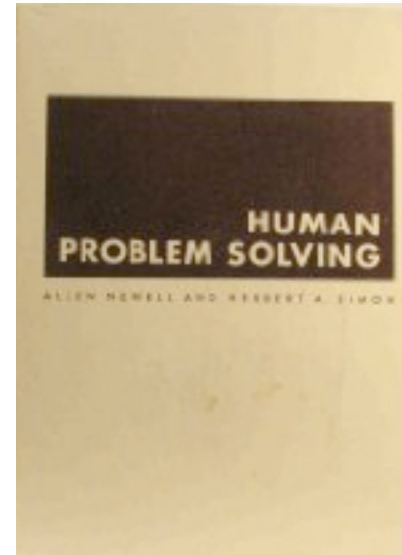
Chapter 3

Some material adopted from notes by Charles R. Dyer, University of Wisconsin-Madison

Today’s topics

- Goal-based agents
- Representing states and actions
- Example problems
- Generic state-space search algorithm
- Specific algorithms
  - Breadth-first search
  - Depth-first search
  - Uniform cost search
  - Depth-first iterative deepening
- Example problems revisited

Big Idea



Allen Newell and Herb Simon developed the *problem space principle* as an AI approach in the late 60s/early 70s

"The rational activity in which people engage to solve a problem can be described in terms of (1) a set of **states** of knowledge, (2) **operators** for changing one state into another, (3) **constraints** on applying operators and (4) **control** knowledge for deciding which operator to apply next."

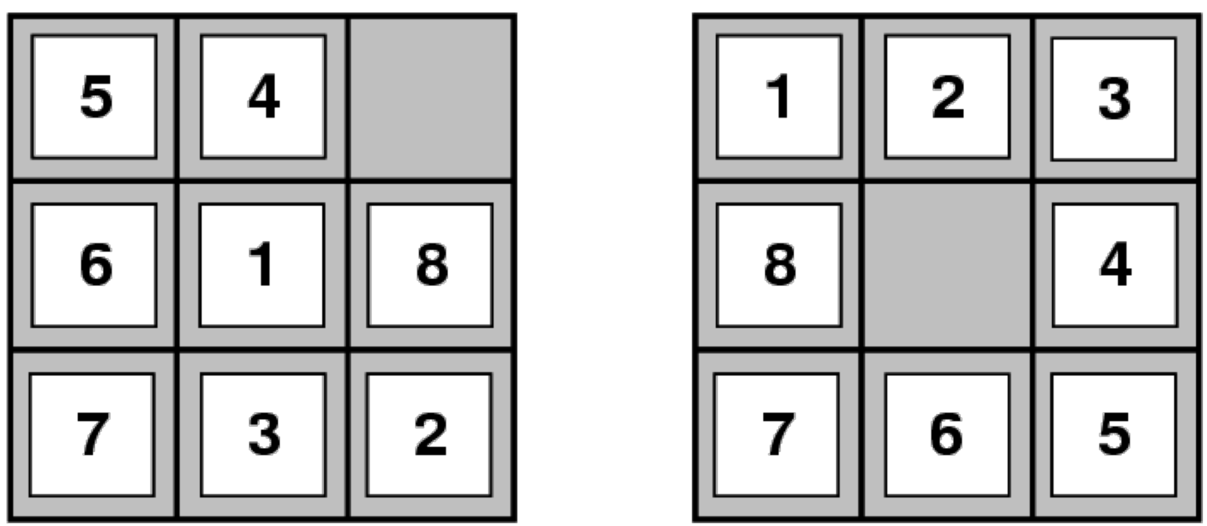
Newell A & *Simon* H A. *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall. 1972.

BTW

- Herb Simon was a polymath who contributed to economics, cognitive science, management, computer science and many other fields
- He was awarded a Nobel Prize in 1978 “for his pioneering research into the decision-making process within economic organizations”
- He is the only computer scientist to have won a Nobel Prize

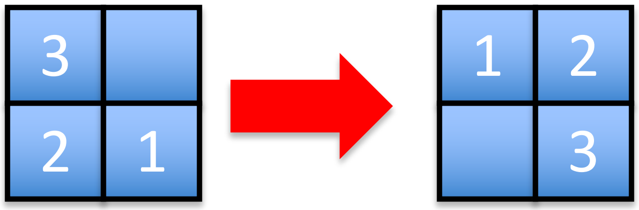
Example: 8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3x3 board, move the tiles in such a way so as to produce a desired goal configuration of the tiles.



Start State Goal State

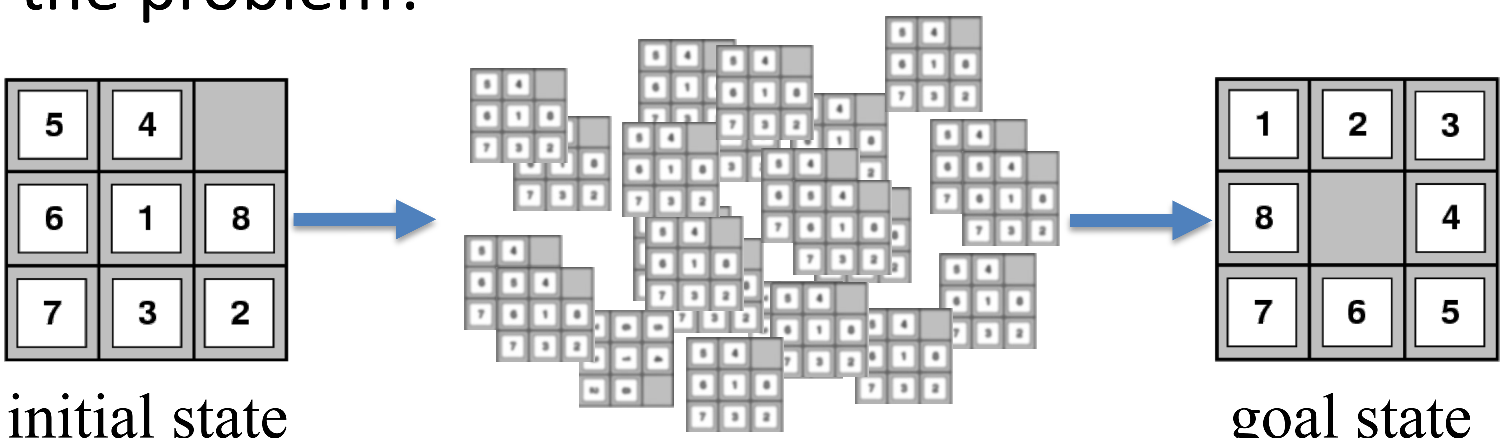
Simpler: 3-Puzzle



Building goal-based agents

We must answer the following questions

- –How do we represent the **state** of the "world"?
- –What is the **goal** and how can we recognize it
- –What are the possible **actions**?
- –What *relevant* information do we encoded to describe the state and available transitions, and solve the problem?



What is the goal to be achieved?

- Can describe a situation we want to achieve, a set of properties that we want to hold, etc.
- Requires defining a **goal test,** so we know what it means to have achieved/satisfied goal
- A hard question, rarely tackled in AI; usually assume system designer or user specifies goal
- Psychologists and motivational speakers stress importance of establishing clear goals as a first step towards solving a problem
- What are your goals???

What are the actions?

- Characterize **primitive actions** for making changes in the world to achieve a goal
- **Deterministic** world: no uncertainty in an action's effects (simple model)
- Given action and description of **current world state**, action completely specifies
  - Whether action ***can*** be applied to the current world (i.e., is it applicable and legal?) and
  - What state ***results*** after action is performed in the current world (i.e., no need for *history* information to compute the next state)

Representing actions

- Actions can be considered as **discrete events** that occur at an **instant of time**, e.g.:

  If "In class" and perform action "go home," then next state is "at home." There's no time where you're neither in class nor at home (i.e., in the state of "going home")

- Number of actions/operators depends on the **representation** used in describing a state
  - 8-puzzle: specify 4 possible moves for each of the 8 tiles, resulting in a total of **4*8=32 operators**
  - Or, we could specify four moves for "blank" square and we only need **4 operators**
- **Representational shift can simplify a problem!**

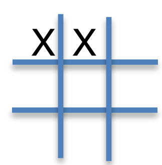
Representing states

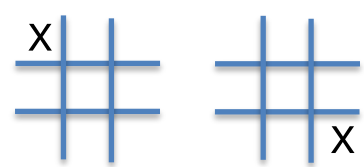
- What information is necessary to describe all relevant aspects to solving the goal?
- **Size of a problem** usually described in terms of possible **number of states**
  - Tic-Tac-Toe has about 3^9 states (19,683≈$2*10^4$)
  - Checkers has about 10^{40} states
  - Rubik's Cube has about 10^{19} states
  - Chess has about 10^{120} states in a typical game
  - Go has $2*10^{170}$
  - Theorem provers may deal with an infinite space
- State space size ≈ solution difficulty

Representing states

- State space size ≈ solution difficulty
- Our estimates were loose upper bounds
- How many legal states does tic-tac-toe really have?

Representing states

- Our estimates were loose upper bounds
- How many **possible, legal** states does tic-tac-toe really have?
- Simple upper bound: nine board cells, each of which can be empty, O or X, so 3^9
- Only 593 states after eliminating
  - impossible states 

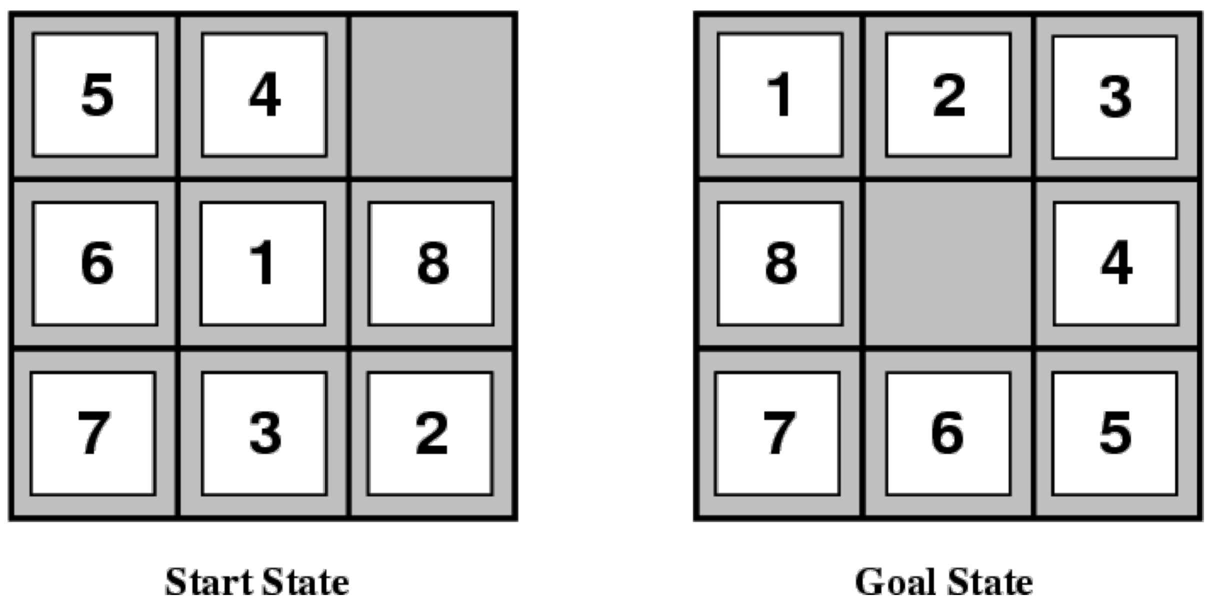
  - Rotations and reflections 

Some example problems

- Toy problems and micro-worlds
  - 8-Puzzle
  - Missionaries and Cannibals
  - Cryptarithmetic
  - Remove 5 Sticks
  - Water Jug Problem
- Real-world problems

8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3x3 board, move the tiles in such a way so as to produce a desired goal configuration of the tiles.



What are the states, goal test, actions?

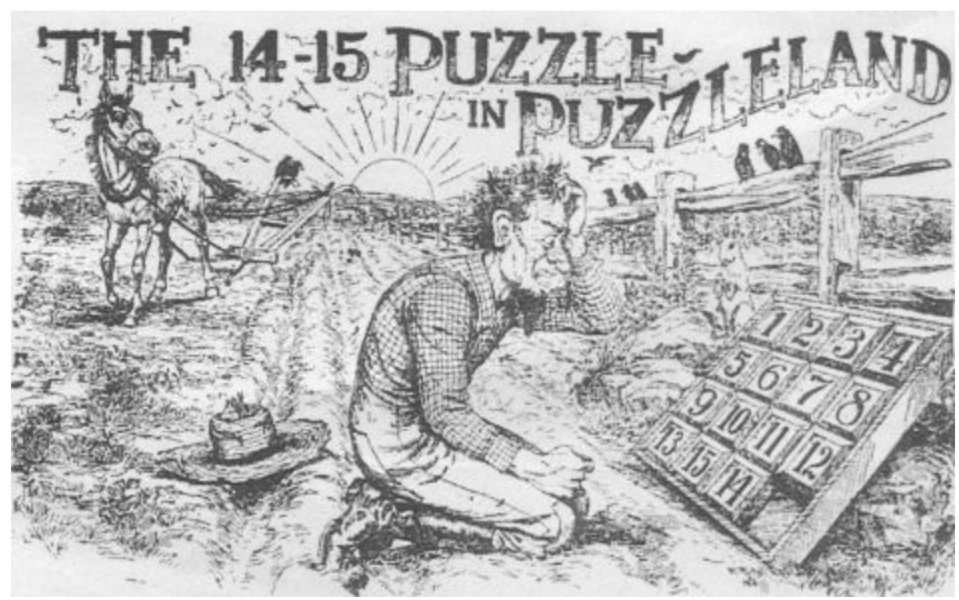
8 puzzle

- **State:** 3x3 array of the tiles on the board
- **Actions:** Move blank square left, right, up or down

  More efficient encoding than one with 4 possible moves for each of 8 distinct tiles
- **Initial State:** A given board configuration
- **Goal:** A given board configuration

15 puzzle

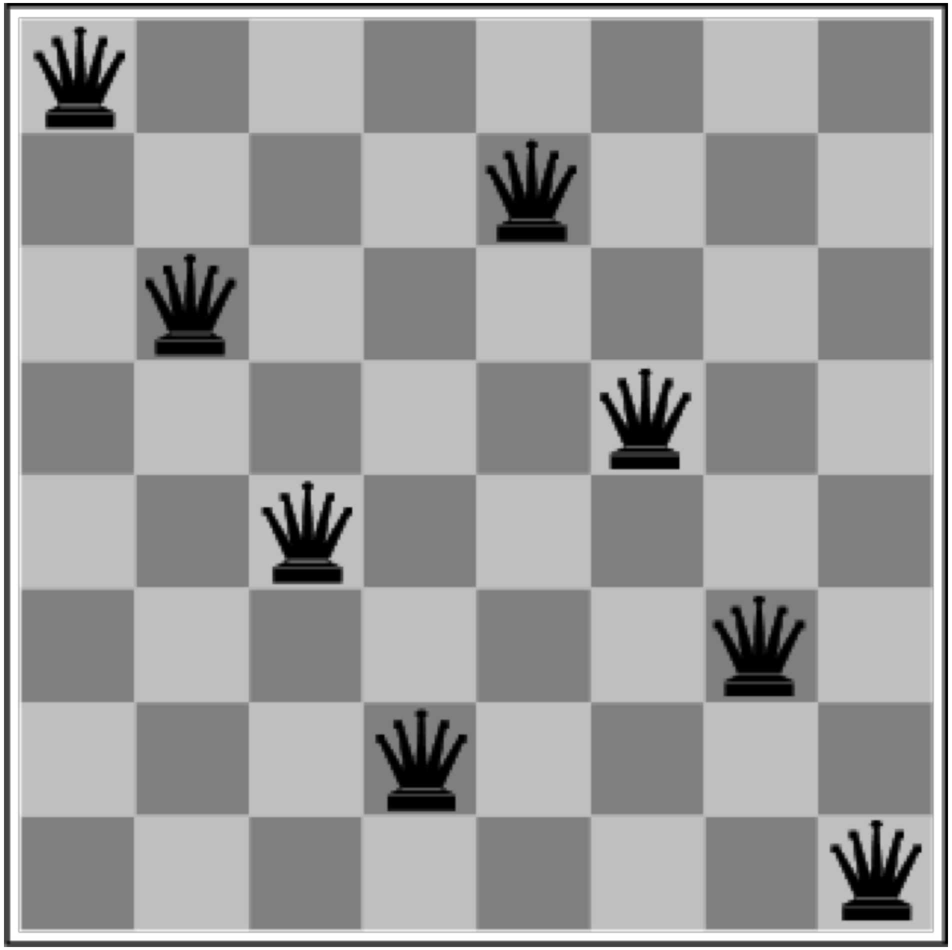
- Popularized, but not invented by, Sam Loyd
- In late 1800s he offered $1000 to all who could find a solution
- He sold many puzzles
- Its states form two disjoint spaces
- There was no path to the solution from his initial state!



The 8-Queens Puzzle

Place eight queens on a chessboard such that no queen attacks any other

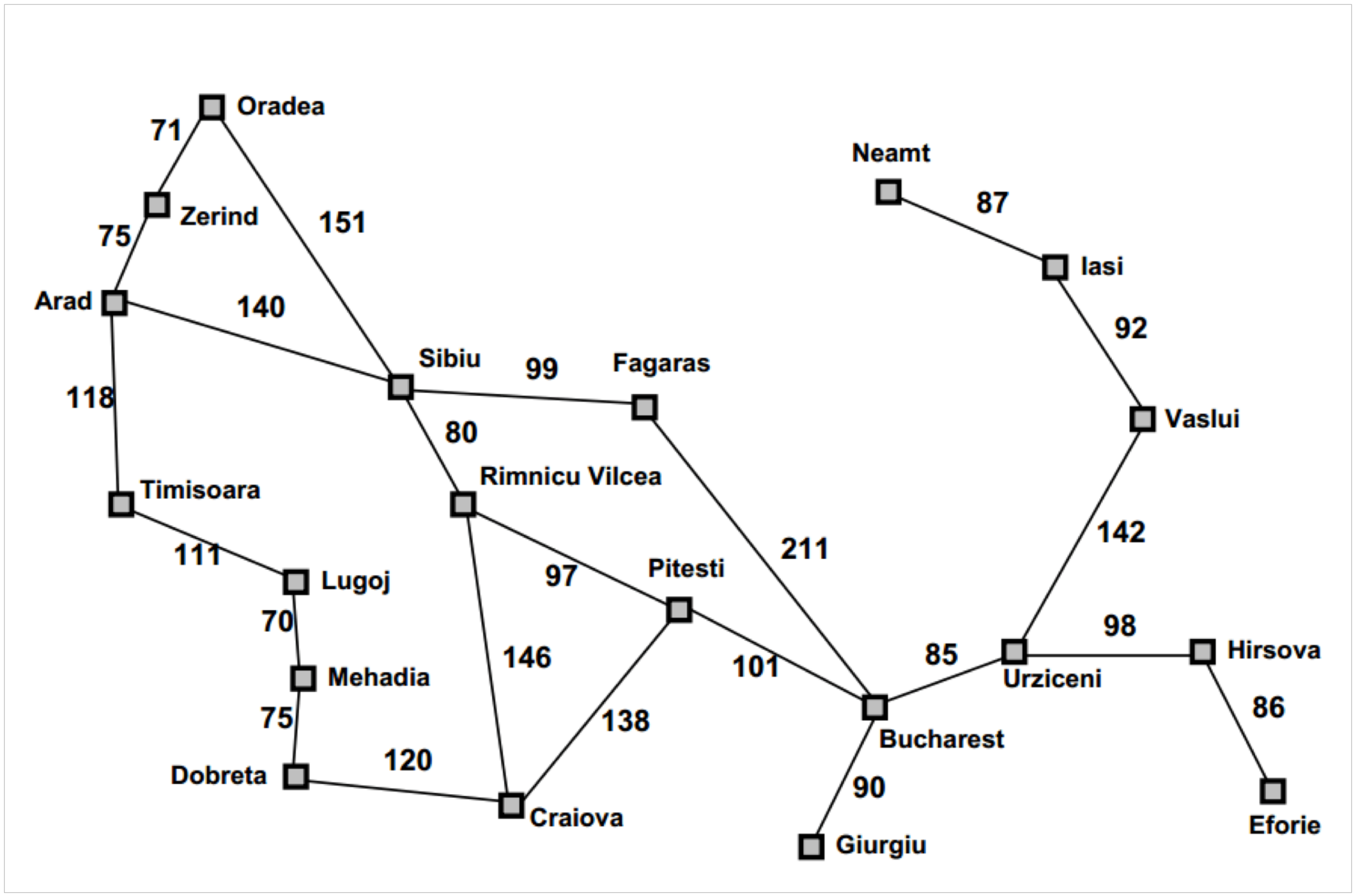
We can generalize the problem to a NxN chessboard



What are the states, goal test, actions?

Route Planning

Find a route from Arad to Bucharest



A simplified map of major roads in Romania used in our text

Example: Water Jug Problem


- Two jugs J1 and J2 with capacity C1 and C2
- Initially J1 has W1 water and J2 has W2 water
  - e.g.: a full 5 gallon jug and an empty 2 gallon jug
- Possible actions:
  - Pour from jug X to jug Y until X empty or Y full
  - Empty jug X onto the floor
- Goal: J1 has G1 water and J2 G2
  - G1 or G0 can be -1 to represent any amount
- E.g.: initially full jugs with capacities 3 and 1 liters, goal is to have 1 liter in each

So…

- How can we represent the states?
- What an initial state
- How do we recognize a goal state
- What are the actions; how can we tell which ones can be performed in a given state; what is the resulting state
- How do we search for a solution from an initial state given a goal state
- What is a solution? The goal state achieved or a path to it?

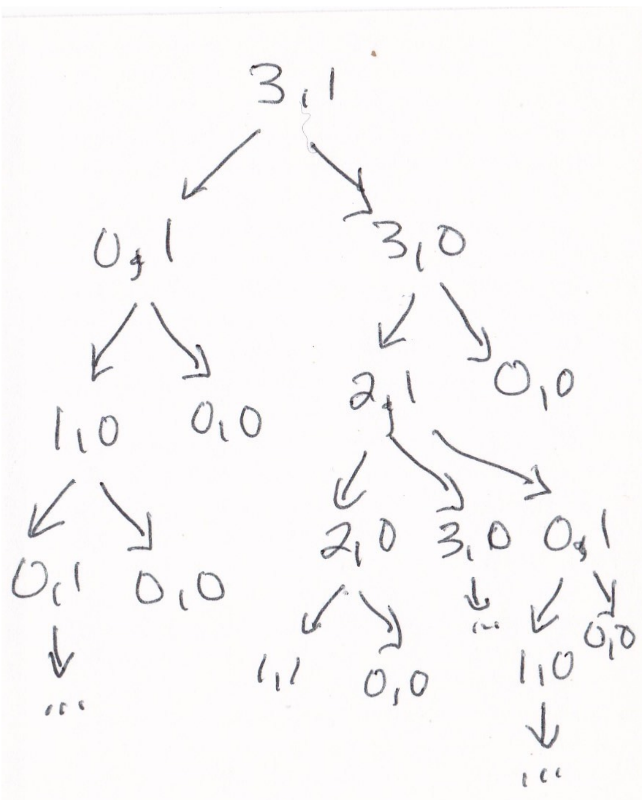
Search in a state space

- Basic idea:
  - Create representation of initial state
  - Try all possible actions & connect states that result
  - Recursively apply process to the new states until we find a solution or dead ends
- We need to keep track of the connections between states and might use a
  - Tree data structure or
  - Graph data structure
- A graph structure is best in general…

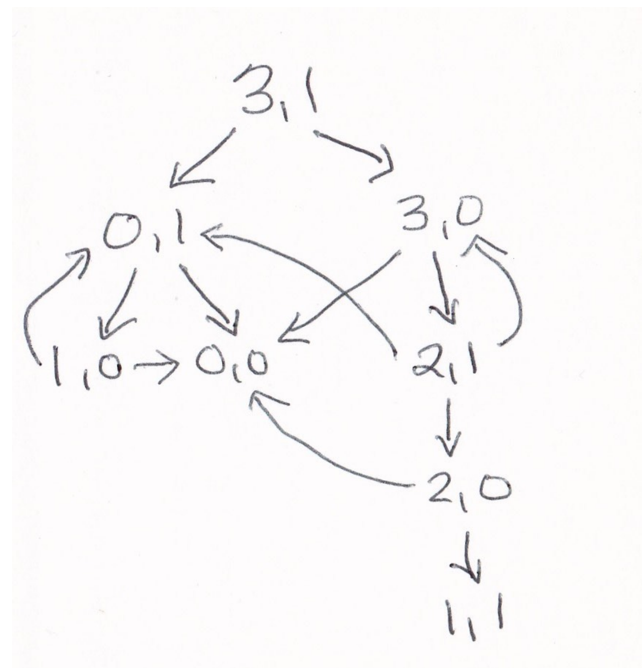
Search in a state space

Consider a water jug problem with a 3-liter and 1-liter jug, an initial state of (3,1) and a goal stage of (1,1)

Tree model of space



Graph model of space



graph model avoids redundancy and loops and is usually preferred

Formalizing search in a state space

- A state space is a **graph** (V, E) where V is a set of **nodes** and E is a set of **arcs**, and each arc is directed from a node to another node
- **Nodes** are data structures with a state description and other info, e.g., node's parent, name of action that generated it from parent, etc.
- **Arcs** are instances of actions. When operator is applied to state at its source node, then resulting state is arc's destination node

Formalizing search in a state space

- Each arc has fixed, positive **cost** associated with it corresponding to the operator cost
  - Simple case: all costs are 1
- Each node has a set of **successor nodes** corresponding to all legal actions that can be applied at node's state
  - **Expanding** a node = generating its successor nodes and adding them and their associated arcs to the graph
- One or more nodes are marked as **start nodes**
- A **goal test** predicate is applied to a state to determine if its associated node is a goal node

Example: Water Jug Problem

- Two jugs J1 and J2 with capacity C1 and C2
- Initially J1 has W1 water and J2 has W2 water
  - e.g.: a full 5 gallon jug and an empty 2 gallon jug
- Possible actions:
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  - G1 or G0 can be -1 to represent any amount

Example: Water Jug Problem

Given full 5 gallon jug and an empty 2 gallon jug, goal is to fill 2 gallon jug with exactly one gallon

- State representation?
  - –General state?
  - –Initial state?
  - –Goal state?
- Possible actions?
  - –Condition?
  - –Resulting state?

Action table

Name	Cond.	Transition	Effect

Example: Water Jug Problem

Given full 5 gallon jug and an empty 2 gallon jug, goal is to fill 2 gallon jug with exactly one gallon

- State = (x,y), where x is water in jug 1 and y is water in jug 2
- Initial State = (5,0)
- Goal State = (-1,1), where -1 means any amount

Action table

Name	Cond.	Transition	Effect
dump1	x>0	$(x,y) \rightarrow (0,y)$	Empty Jug 1
dump2	y>0	$(x,y) \rightarrow (x,0)$	Empty Jug 2
pour_1_2	x>0 & y<C2	$(x,y) \rightarrow (x-D,y+D)$ $D = min(x,C2-y)$	Pour from Jug 1 to Jug 2
pour_2_1	y>0 & X<C1	$(x,y) \rightarrow (x+D,y-D)$ $D = min(y,C1-x)$	Pour from Jug 2 to Jug 1

Class Exercise

- Representing a 2x2 Sudoku puzzle as a search space
- Fill in the grid so that every row, every column, and every 2x2 box contains the digits 1 through 4
  - What are the states?
  - What are the actions?
  - What are the constraints on actions?
  - What is the description of the goal state?

	3		
			1
3			
		2	

Formalizing search (3)

- **Solution:** sequence of actions associated with a path from a start node to a goal node
- **Solution cost:** sum of the arc costs on the solution path
  - If all arcs have same (unit) cost, then solution cost is just the length of solution (number of steps / state transitions)
  - Algorithms generally require that arc costs cannot be negative (why?)

Formalizing search (4)

- **State-space search:** searching through state space for solution by **making explicit** a sufficient portion of an **implicit** state-space graph to find a goal node
  - Can’t materializing whole space for large problems
  - Initially V={S}, where S is the start node, E={}
  - On expanding S, its successor nodes are generated and added to V and associated arcs added to E
  - Process continues until a goal node is found
- Nodes represent a *partial solution* path (+ cost of partial solution path) from S to the node
  - From a node there may be many possible paths (and thus solutions) with this partial path as a prefix

State-space search algorithm

;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; general-search returns either a goal node or failure

```
function general-search (problem, QUEUEING-FUNCTION)
  nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
                problem.OPERATORS))
 end
```

;; Note: The goal test is NOT done when nodes are generated
;; Note: This algorithm does not detect loops

Key procedures to be defined

- EXPAND
  - Generate all successor nodes of a given node, adding them to the graph
- GOAL-TEST
  - Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
  - Used to maintain a ranked list of nodes that are candidates for expansion

Bookkeeping

Typical node data structure includes:

- State at this node
- Parent node(s)
- Action(s) applied to get to this node
- Depth of this node (# of actions on shortest known path from initial state)
- Cost of path (sum of action costs on best path from initialstate)

Some issues

- Search process constructs a search tree/graph, where
  - **root** is initial state and
  - **leaf nodes** are nodes
    - not yet expanded (i.e., in list "nodes") or
    - having no successors (i.e., they're *deadends* because no operators were applicable and yet they are not goals)
- Search tree may be infinite due to loops; even graph may be infinite for some problems
- Solution is a *path* or a *node*, depending on problem.
  - E.g., in cryptarithmetic return a node; in 8-puzzle, a path
- Changing definition of the QUEUEING-FUNCTION leads to different search strategies

Uninformed vs. informed search

Uninformed search strategies (blind search)

- Use no information about likely “direction” of goal node(s)
- Methods: breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional

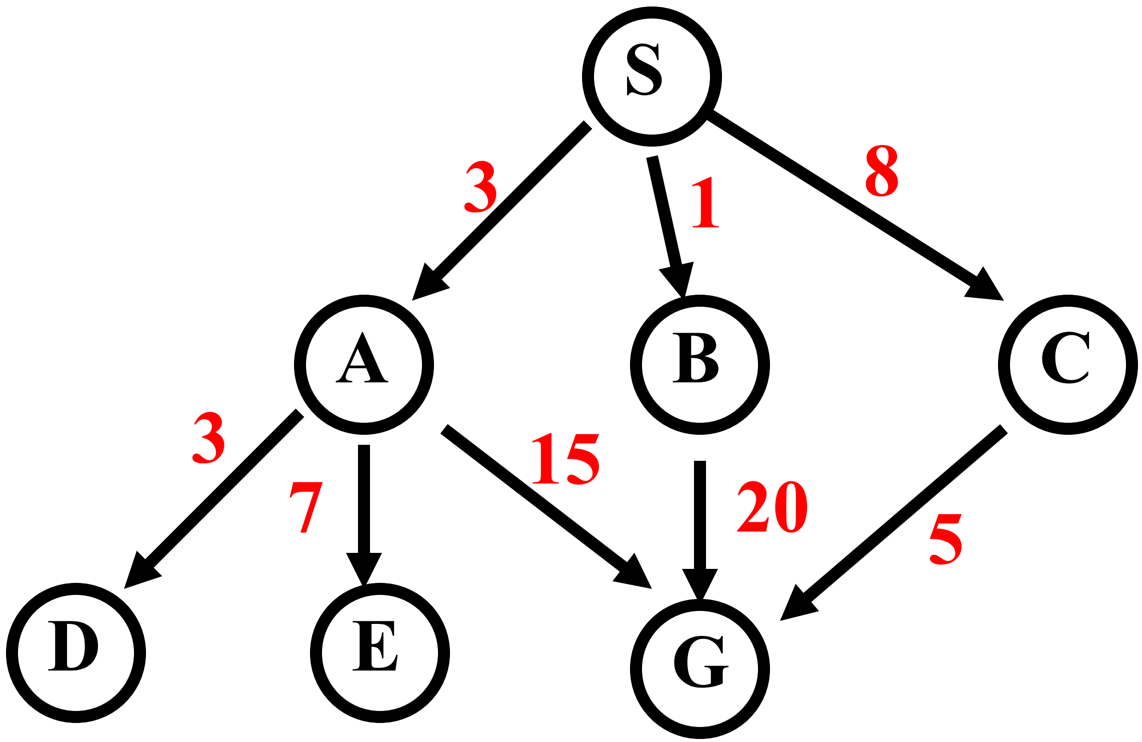
Informed search strategies (heuristic search)

- Use information about domain to (try to) (usually) head in the general direction of goal node(s)
- Methods: hill climbing, best-first, greedy search, beam search, algorithm A, algorithm A*

Evaluating search strategies

- **Completeness**
  – Guarantees finding a solution whenever one exists
- **Time complexity** (worst or average case)
  – Usually measured by *number of nodes expanded*
- **Space complexity**
  – Usually measured by maximum size of graph/tree during the search
- **Optimality/Admissibility**
  – If a solution is found, is it **guaranteed** to be an optimal one, i.e., one with minimum cost

Example of uninformed search strategies



Consider this search space where S is the start node and G is the goal. Numbers are arc costs.

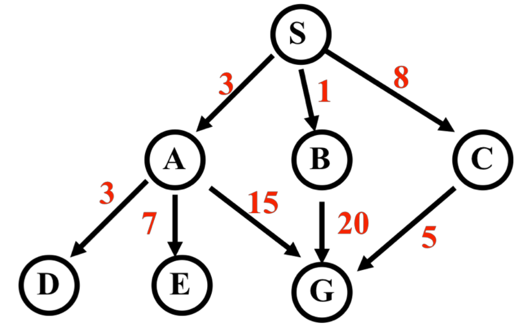
Classic uninformed search methods

- The four classic uninformed search methods
  - Breadth first search (BFS)
  - Depth first search (DFS)
  - Uniform cost search *(generalization of BFS)*
  - Iterative deepening *(blend of DFS and BFS)*
- To which we can add another technique
  - Bi-directional search *(hack on BFS)*

Breadth-First Search

- Enqueue nodes in **FIFO** (first-in, first-out) order
- **Complete**
- **Optimal** (i.e., admissible) finds shorted path, which is optimal if all operators have same cost
- **Exponential time and space complexity**, $O(b^d)$, where d is depth of solution and b is branching factor (i.e., # of children)
- Takes a **long time to find solutions** with large number of steps because must look at all shorter length possibilities first

Breadth-First Search
weighted arcs



Expanded node	Nodes list (aka Fringe)
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ B^1 C^8 D^6 E^{10} G^{18} }
B^1	{ C^8 D^6 E^{10} G^{18} G^{21} }
C^8	{ D^6 E^{10} G^{18} G^{21} G^{13} }
D^6	{ E^{10} G^{18} G^{21} G^{13} }
E^{10}	{ G^{18} G^{21} G^{13} }
G^{18}	{ G^{21} G^{13} }

Notation

G^{18}

G is node; 18 is cost of shortest known path from start node S

Note: we typically don't check for goal until we expand node
Solution path found is S A G , cost 18
Number of nodes expanded (including goal node) = 7

Breadth-First Search

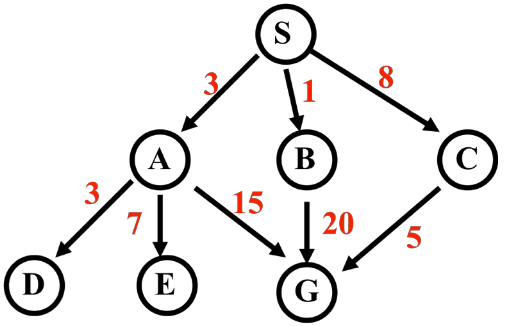
Long time to find solutions with many steps: we must look at all shorter length possibilities first

- Complete search tree of depth d where non-leaf nodes have b children has $1 + b + b^2 + ... + b^d = (b^{(d+1)} - 1)/(b-1)$ nodes = $0(b^d)$
- Tree of depth 12 with branching 10 has more than a trillion nodes
- If BFS expands 1000 nodes/sec and nodes uses 100 bytes, then it may take 35 years to run and uses 111 terabytes of memory!

Depth-First (DFS)

- Enqueue nodes on nodes in **LIFO** (last-in, first-out) order, i.e., use stack data structure to order nodes
- **May not terminate** w/o *depth bound*, i.e., ending search below fixed depth D (depth-limited search)
- **Not complete** (with or w/o cycle detection, with or w/o a cutoff depth)
- **Exponential time**, $O(b^d)$, but **linear space**, $O(bd)$
- Can find **long solutions quickly** if lucky (and **short solutions slowly** if unlucky!)
- On reaching deadend, can only back up one level at a time even if problem occurs because of a bad choice at top of tree

Depth-First Search



Expanded node	Nodes list
	$\{ S^0 \}$
S^0	$\{ A^3\ B^1\ C^8 \}$
A^3	$\{ D^6\ E^{10}\ G^{18}\ B^1\ C^8 \}$
D^6	$\{ E^{10}\ G^{18}\ B^1\ C^8 \}$
E^{10}	$\{ G^{18}\ B^1\ C^8 \}$
G^{18}	$\{ B^1\ C^8 \}$

Solution path found is S A G, cost 18

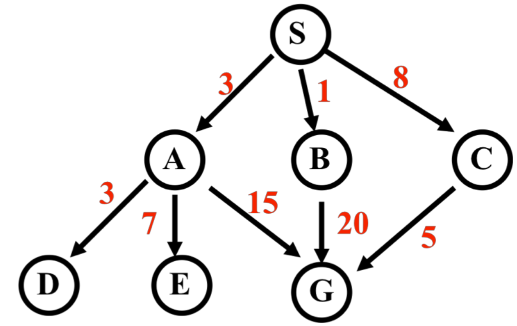
Number of nodes expanded (including goal node) = 5

Uniform-Cost Search (UCS)

- Enqueue nodes by **path cost**. i.e., let g(n) = cost of path from *start* to current node *n*. Sort nodes by increasing value of g(n).
- Also called *Dijkstra's Algorithm*, similar to *Branch and Bound Algorithm* from operations research
- **Complete (*)**
- **Optimal/Admissible** (*)

  Depends on goal test being applied *when node is removed from nodes list*, not when its parent node is expanded & node first generated
- **Exponential time and space complexity**, $O(b^d)$

Uniform-Cost Search



Expanded node	Nodes list
	{ S^0 }
S^0	{ B^1 A^3 C^8 }
B^1	{ A^3 C^8 G^{21} }
A^3	{ D^6 C^8 E^{10} G^{18} G^{21} }
D^6	{ C^8 E^{10} G^{18} G^{21} }
C^8	{ E^{10} G^{13} G^{18} G^{21} }
E^{10}	{ G^{13} G^{18} G^{21} }
G^{13}	{ G^{18} G^{21} }

Solution path found is S C G, cost 13

Number of nodes expanded (including goal node) = 7

Depth-First Iterative Deepening (DFID)

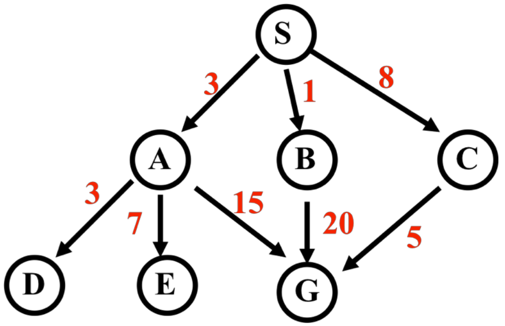
- Do DFS to depth 0, then (if no solution) DFS to depth 1, etc.
- Usually used with a tree search
- **Complete**
- **Optimal/Admissible** if all operators have unit cost, else finds shortest solution (like BFS)
- Time complexity a bit worse than BFS or DFS

  Nodes near top of search tree generated many times, but since almost all nodes are near tree bottom, worst case time complexity still exponential, $O(b^d)$

Depth-First Iterative Deepening (DFID)

- If branching factor is b and solution is at depth d, then nodes at depth d are generated once, nodes at depth d-1 are generated twice, etc.
  - Hence $b^d + 2b^{(d-1)} + ... + db <= b^d / (1 - 1/b)^2 = O(b^d)$.
  - If b=4, worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth d (in worst case)
- **Linear space complexity**, O(bd), like DFS
- Has advantages of BFS (completeness) and DFS (i.e., limited space, finds longer paths quickly)
- Preferred for **large state spaces** where **solution depth is unknown**

How they perform



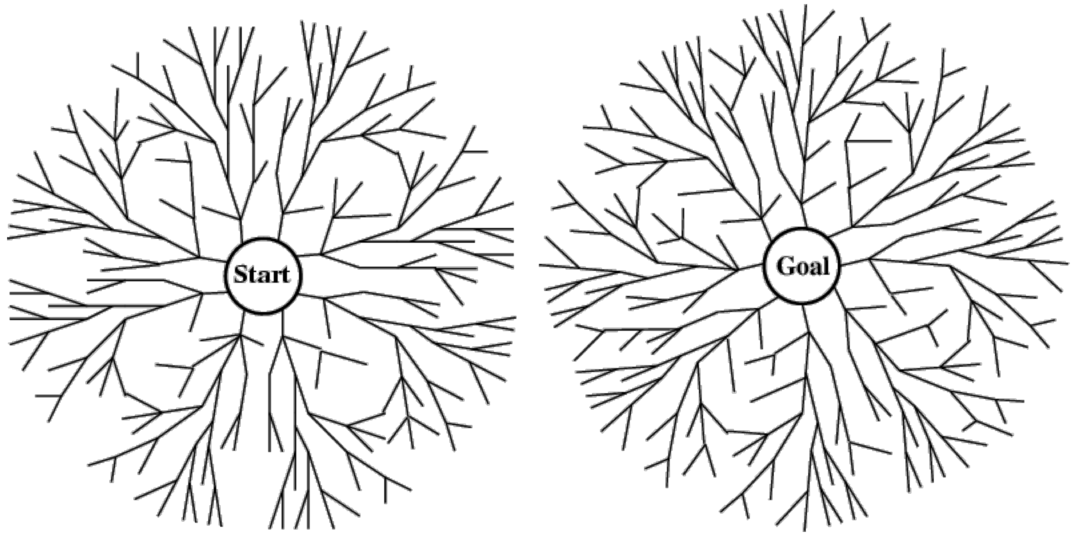
- **Depth-First Search:**
  - 4 Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Breadth-First Search**:
  - 7 Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - 7 Expanded nodes: S A D B C E G
  - Solution found: S C G (cost 13)

  Only uninformed search that worries about costs
- **Iterative-Deepening Search**:
  - 10 nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)

Searching Backward from Goal

- Usually a successor function is reversible
  - i.e., can generate a node's predecessors in graph
- If we know a single goal (rather than a goal's properties), we could search backward to the initial state
- It might be more efficient
  - Depends on whether the graph fans in or out

Bi-directional search



- Alternate searching from the start state toward the goal and from the goal state toward the start
- Stop when the frontiers intersect
- Works well only when there are unique start & goal states
- Requires ability to generate "predecessor" states
- Can (sometimes) lead to finding a solution more quickly

Comparing Search Strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes