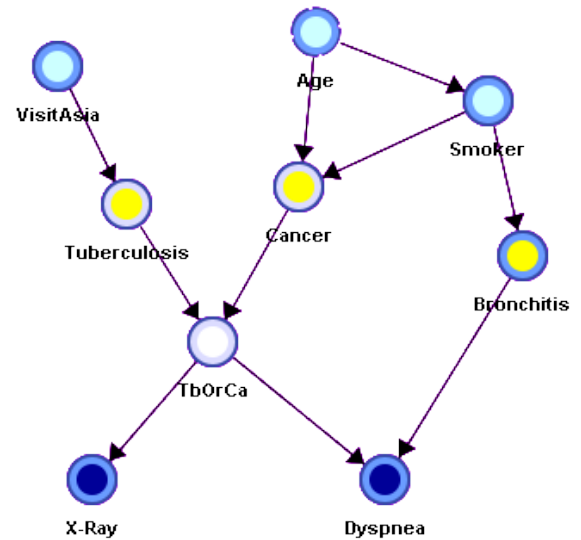


Reasoning with Bayesian Belief Networks

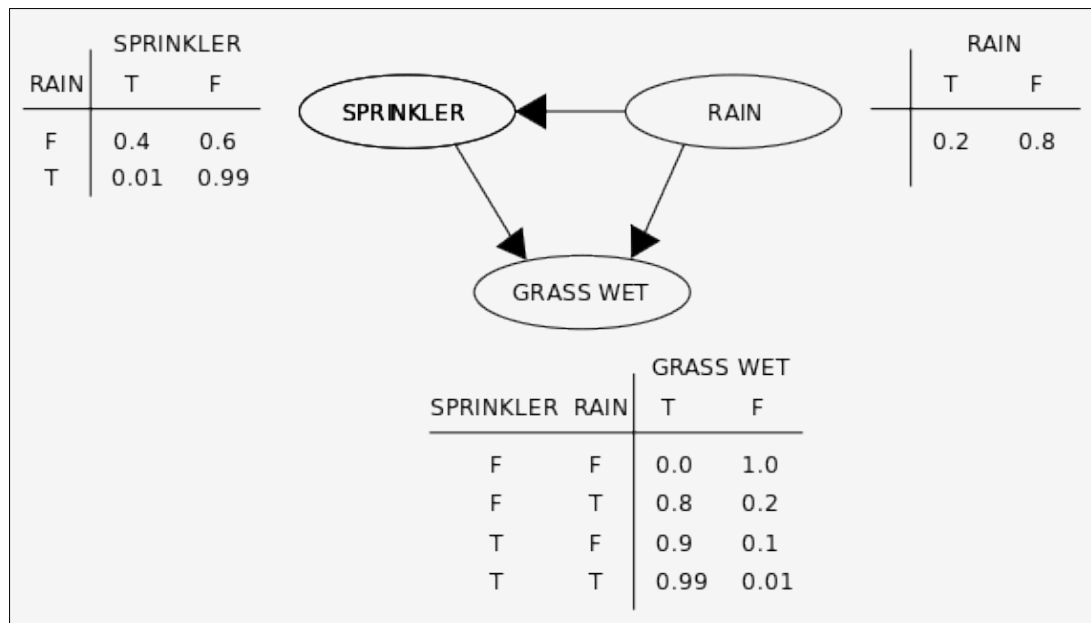


Overview

- Bayesian Belief Networks (BBNs) can reason with networks of propositions and associated probabilities
- Useful for many AI problems
 - Diagnosis
 - Expert systems
 - Planning
 - Learning

BBN Definition

- AKA Bayesian Network, Bayes Net
- A graphical model (as a DAG) of probabilistic relationships among a set of random variables
- Links represent direct influence of one variable on another



Recall Bayes Rule

$$P(H, E) = P(H | E)P(E) = P(E | H)P(H)$$

$$P(H | E) = \frac{P(E | H)P(H)}{P(E)}$$

Note the symmetry: we can compute the probability of a hypothesis given its evidence and vice versa

Simple Bayesian Network

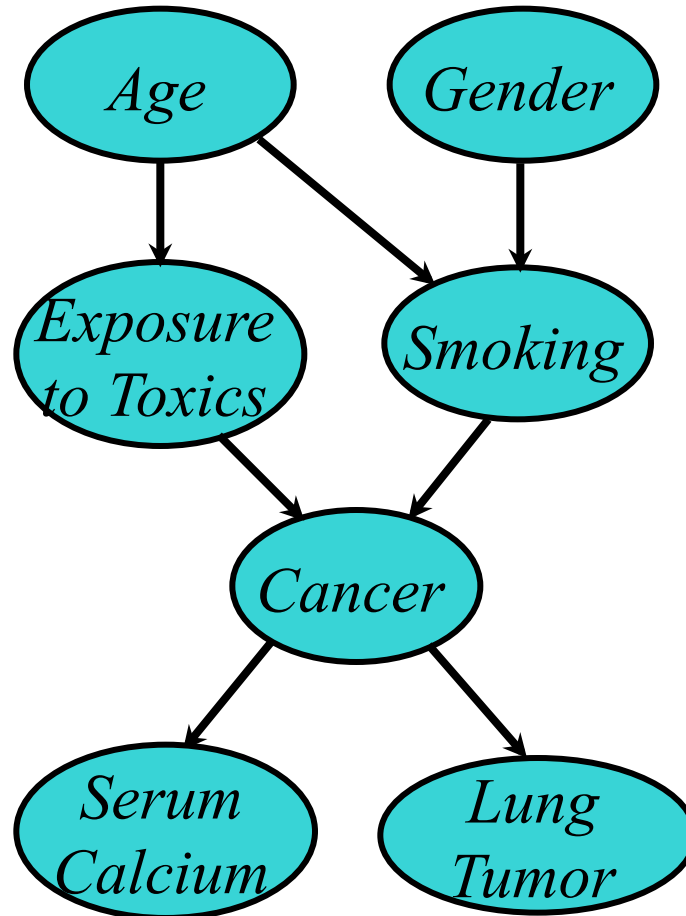


$P(S=no)$	0.80
$P(S=light)$	0.15
$P(S=heavy)$	0.05

$C \in \{none, benign, malignant\}$

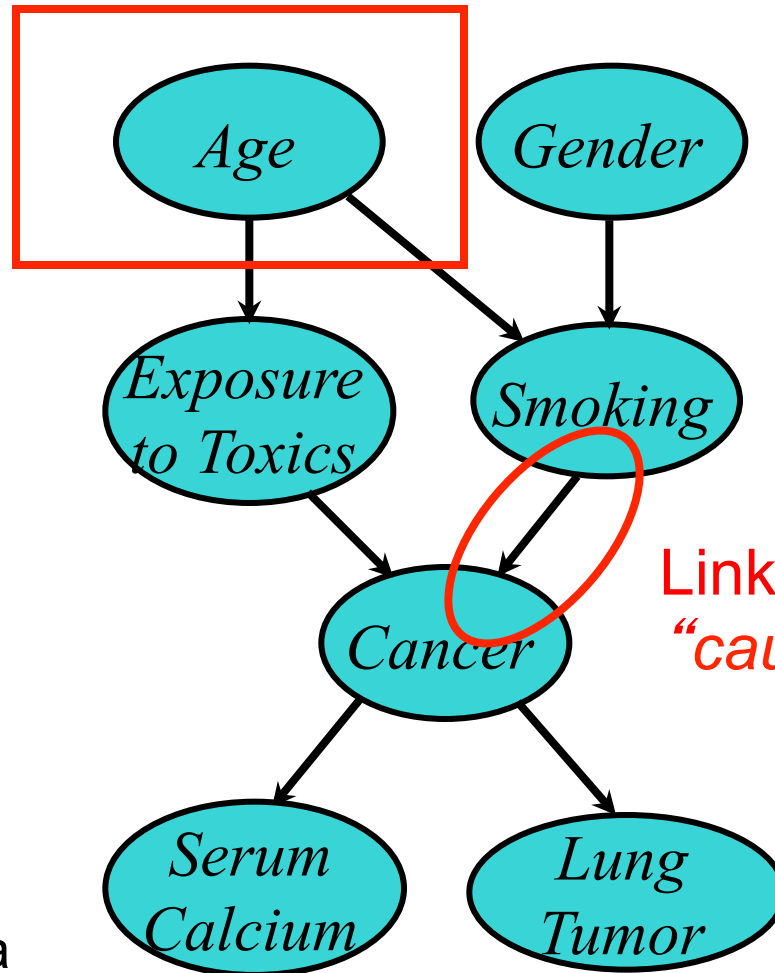
$Smoking =$	no	$light$	$heavy$
$P(C=none)$	0.96	0.88	0.60
$P(C=benign)$	0.03	0.08	0.25
$P(C=malig)$	0.01	0.04	0.15

More Complex Bayesian Network



More Complex Bayesian Network

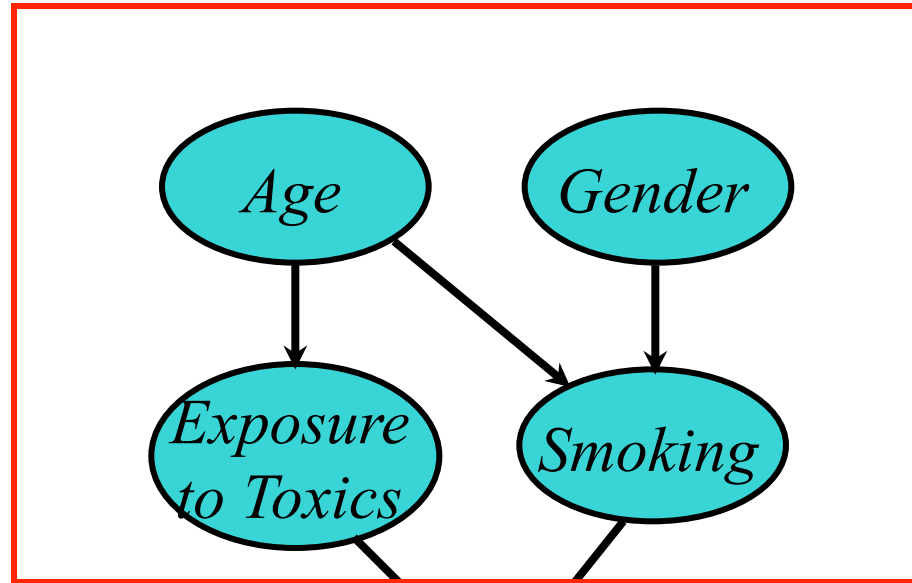
Nodes represent variables



Links represent "causal" relations

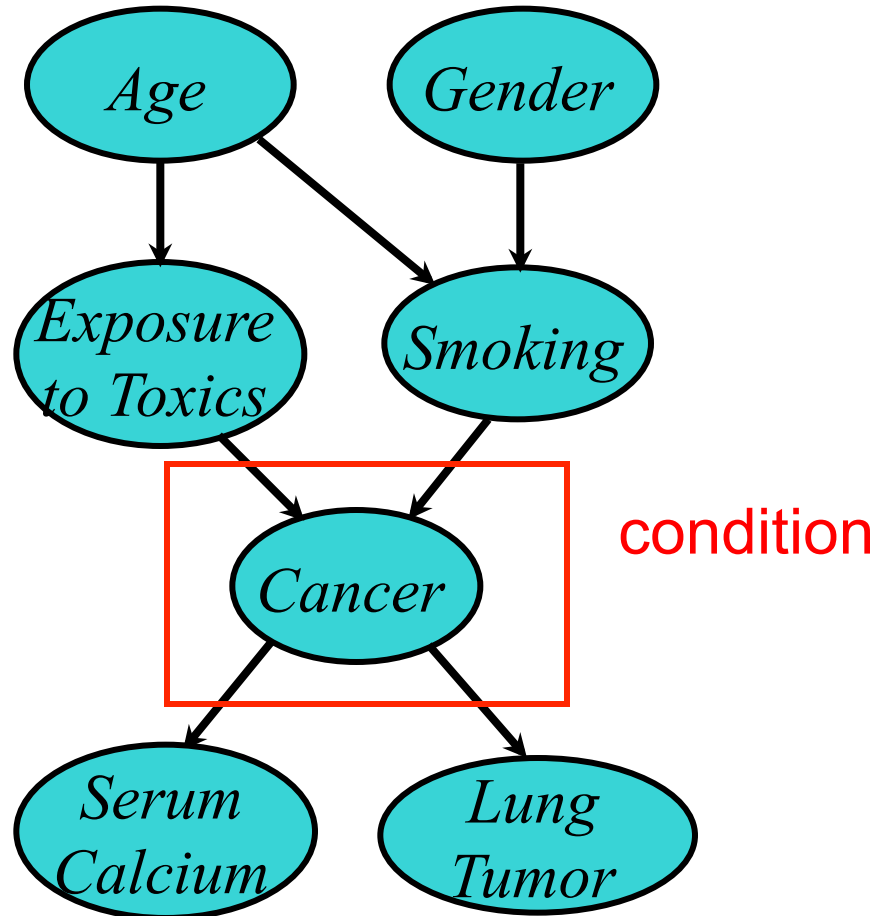
- Does gender cause smoking?
- Influence might be a more appropriate term

More Complex Bayesian Network

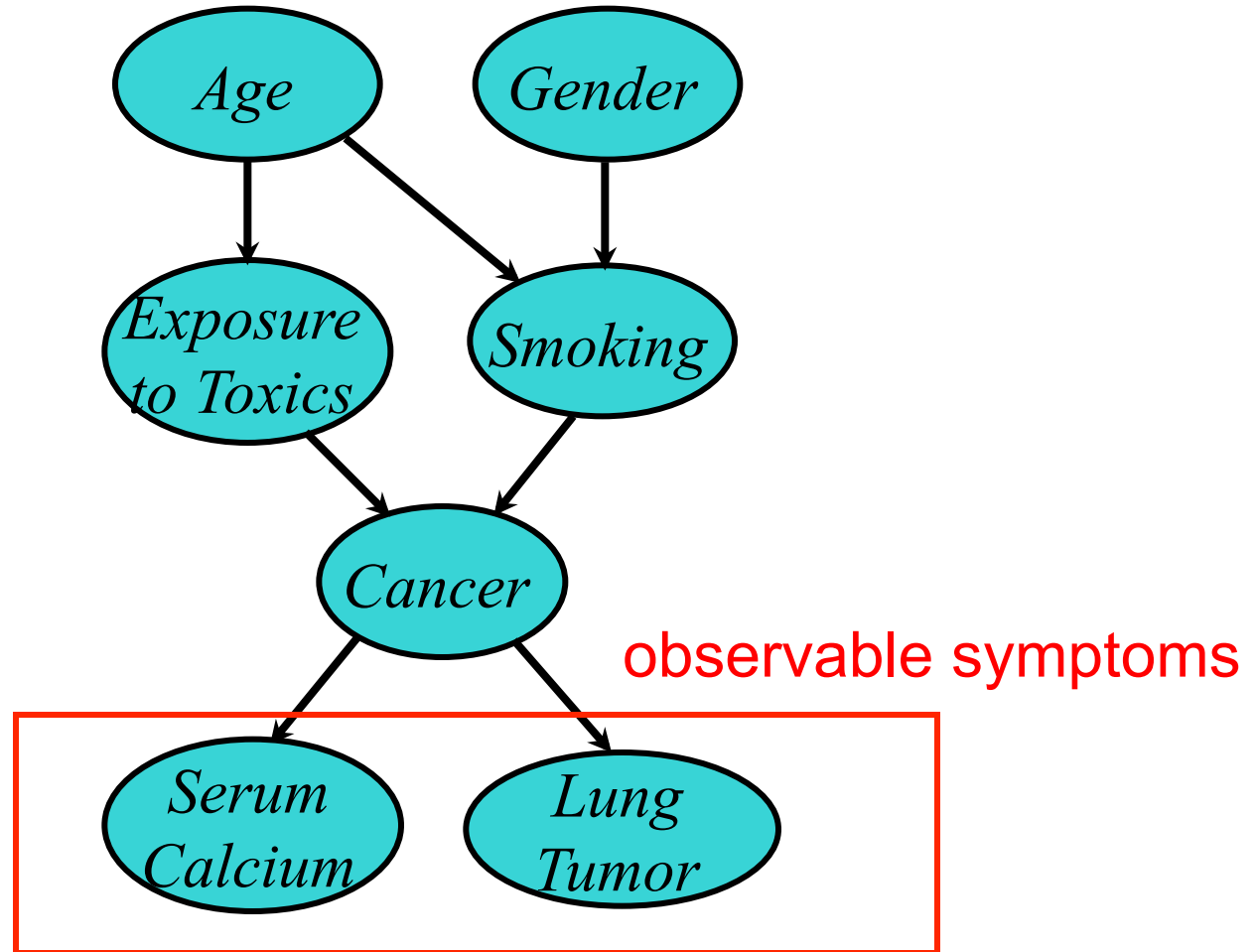


predispositions

More Complex Bayesian Network



More Complex Bayesian Network



Independence

Age

Gender

Age and Gender are independent.

$$P(A, G) = P(G) * P(A)$$

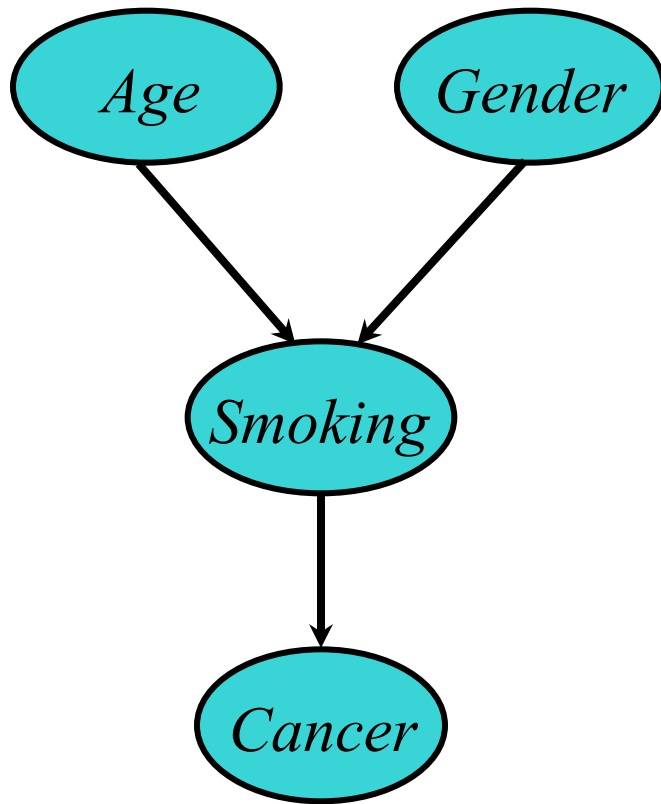
$$P(A | G) = P(A)$$

$$P(G | A) = P(G)$$

$$P(A, G) = P(G|A) P(A) = P(G)P(A)$$

$$P(A, G) = P(A|G) P(G) = P(A)P(G)$$

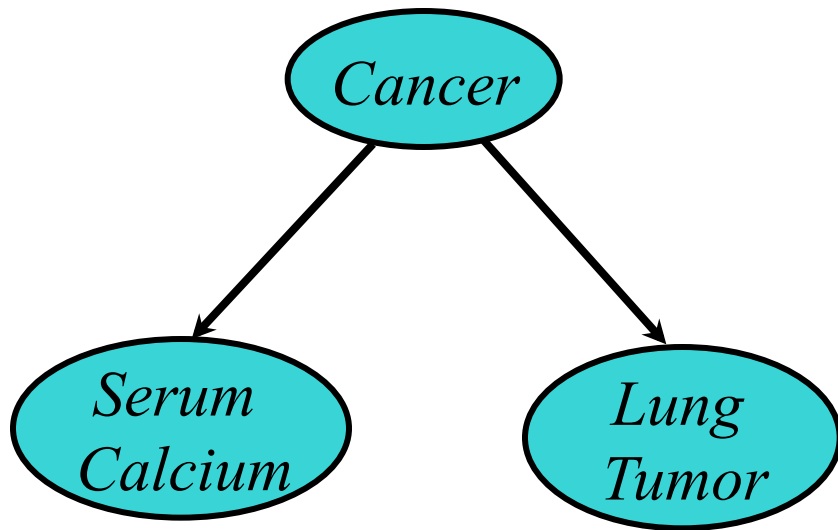
Conditional Independence



Cancer is independent of *Age* and *Gender* given *Smoking*

$$P(C | A, G, S) = P(C | S)$$

Conditional Independence: Naïve Bayes



Serum Calcium and Lung Tumor are dependent

Serum Calcium is independent of Lung Tumor, given Cancer

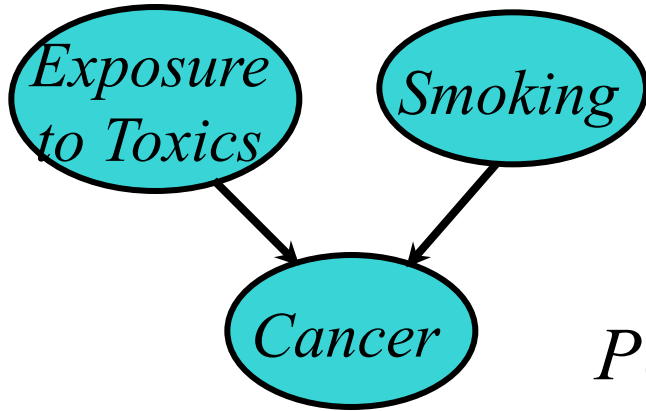
$$P(L | SC, C) = P(L|C)$$
$$P(SC | L, C) = P(SC|C)$$

Naïve Bayes assumption: evidence (e.g., symptoms) is independent given the disease. This make it easy to combine evidence

Explaining Away

Exposure to Toxics and Smoking are independent

*Exposure to Toxics is **dependent** on Smoking, given Cancer*

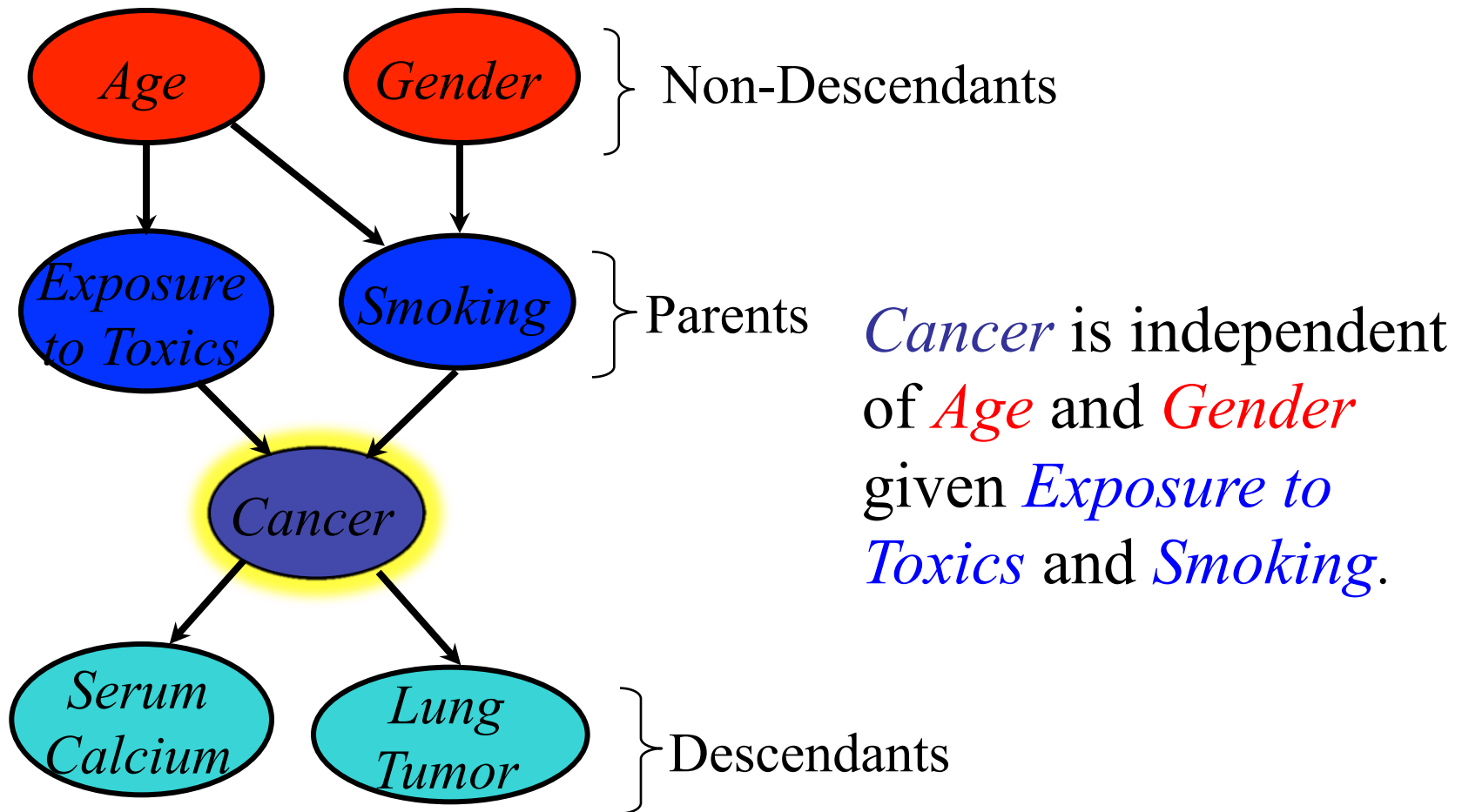


$$P(E=heavy \mid C=malignant) > P(E=heavy \mid C=malignant, S=heavy)$$

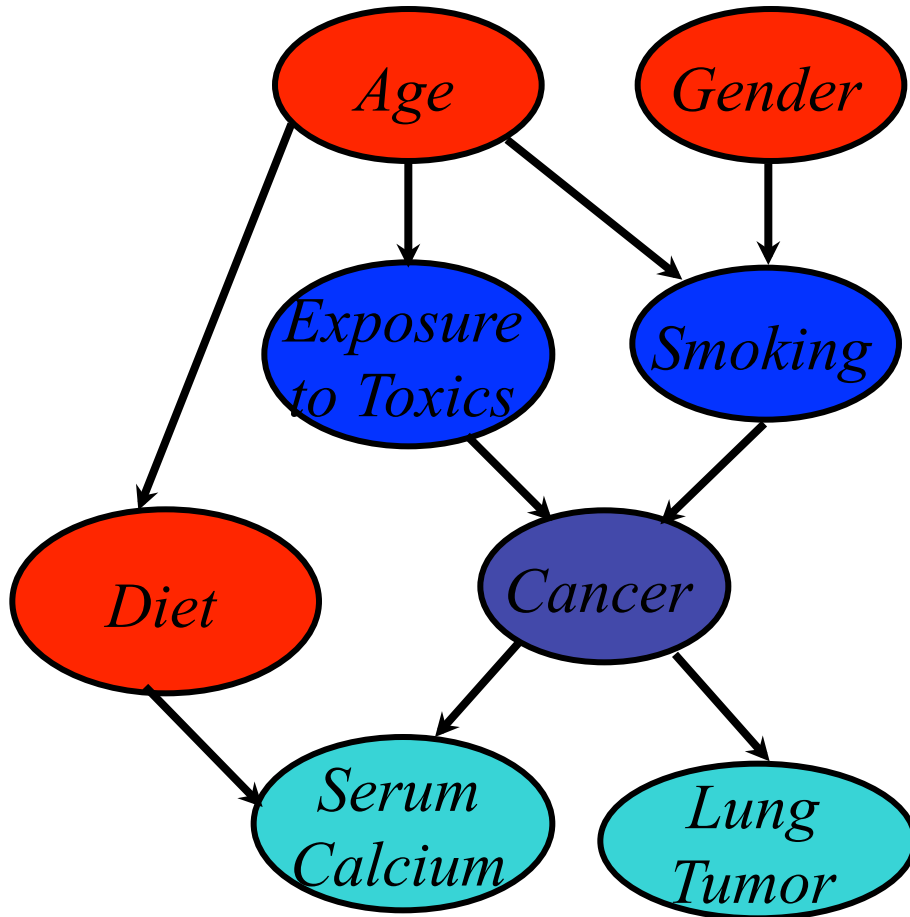
- *Explaining away*: reasoning pattern where confirmation of one cause reduces need to invoke alternatives
- Essence of [Occam's Razor](#) (prefer hypothesis with fewest assumptions)
- Relies on independence of causes

Conditional Independence

A variable (node) is conditionally independent of its non-descendants given its parents



Another non-descendant



A variable is conditionally independent of its non-descendants given its parents

Cancer is independent of *Diet* given *Exposure to Toxics* and *Smoking*

BBN Construction

The knowledge acquisition process for a BBN involves three steps

KA1: Choosing appropriate variables

KA2: Deciding on the network structure

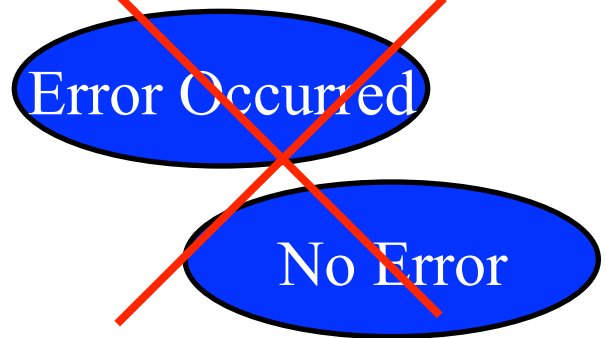
KA3: Obtaining data for the conditional probability tables

KA1: Choosing variables

- Variable values can be integers, reals or enumerations
- Variable should have collectively exhaustive, mutually exclusive values

$$x_1 \vee x_2 \vee x_3 \vee x_4$$

$$\neg (x_i \wedge x_j) \quad i \neq j$$



- They should be values, not probabilities

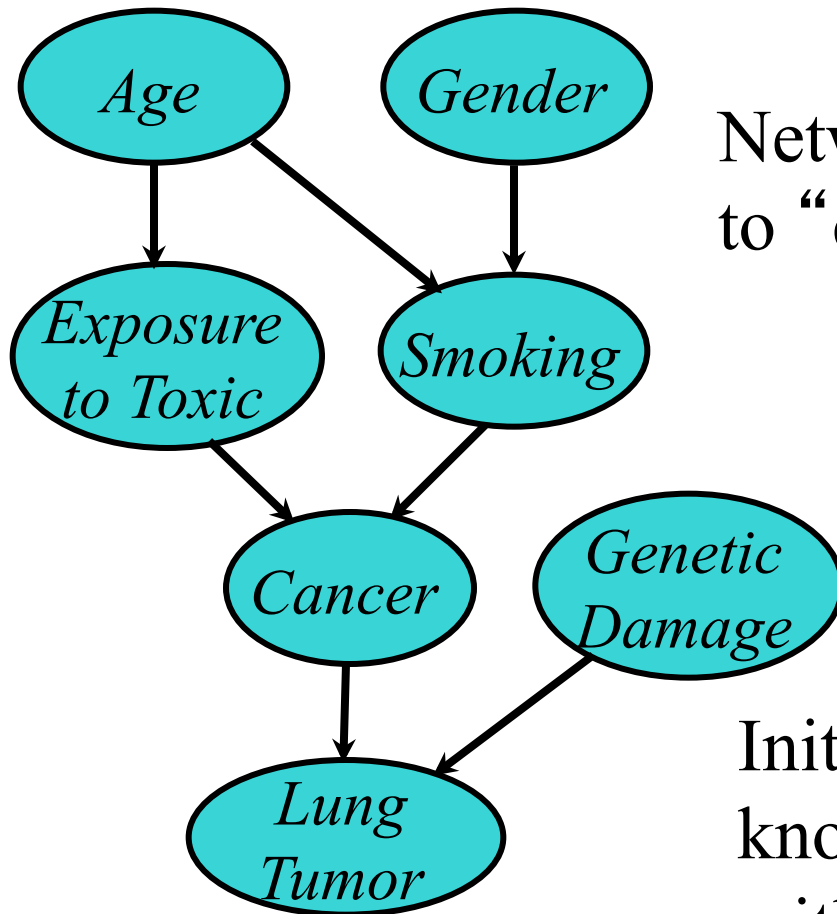


Heuristic: Knowable in Principle

Example of good variables

- Weather: {Sunny, Cloudy, Rain, Snow}
- Gasoline: Cents per gallon {0,1,2...}
- Temperature: { $\geq 100^{\circ}\text{F}$, $< 100^{\circ}\text{F}$ }
- User needs help on Excel Charting: {Yes, No}
- User's personality: {dominant, submissive}

KA2: Structuring



Network structure corresponding to “causality” is usually good.

Initially this uses the designer’s knowledge but can be checked with data

KA3: The Numbers

- For each variable we have a table of probability of its value for values of its parents
- For variables w/o parents, we have prior probabilities

$S \in \{no, light, heavy\}$

$C \in \{none, benign, malignant\}$



smoking priors	
no	0.80
light	0.15
heavy	0.05

	smoking		
cancer	no	light	heavy
none	0.96	0.88	0.60
benign	0.03	0.08	0.25
malignant	0.01	0.04	0.15

KA3: The numbers

- Second decimal usually doesn't matter
- Relative probabilities are important

E-Arousal	Fast	Normal	Slow
Passive	.20	.28	.52
Neutral	.33	.33	.33
Excited	.56	.27	.16

- Zeros and ones are often enough
- Order of magnitude is typical: 10^{-9} vs 10^{-6}
- Sensitivity analysis can be used to decide accuracy needed

Three kinds of reasoning

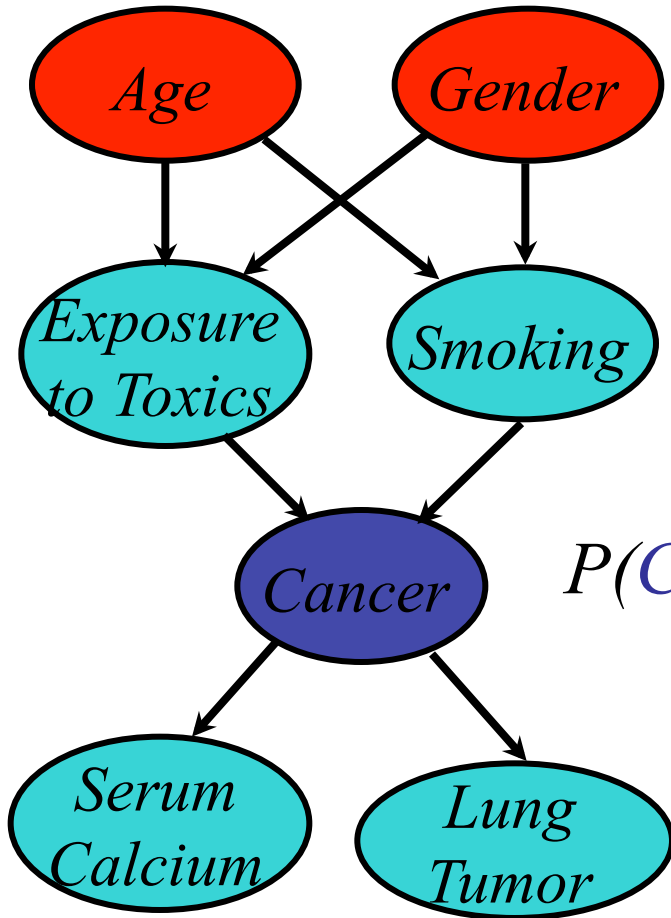
BBNs support three main kinds of reasoning:

- **Predicting** conditions given predispositions
- **Diagnosing** conditions given symptoms (and predisposing)
- **Explaining** a condition by one or more predispositions

To which we can add a fourth:

- **Deciding** on an action based on probabilities of the conditions

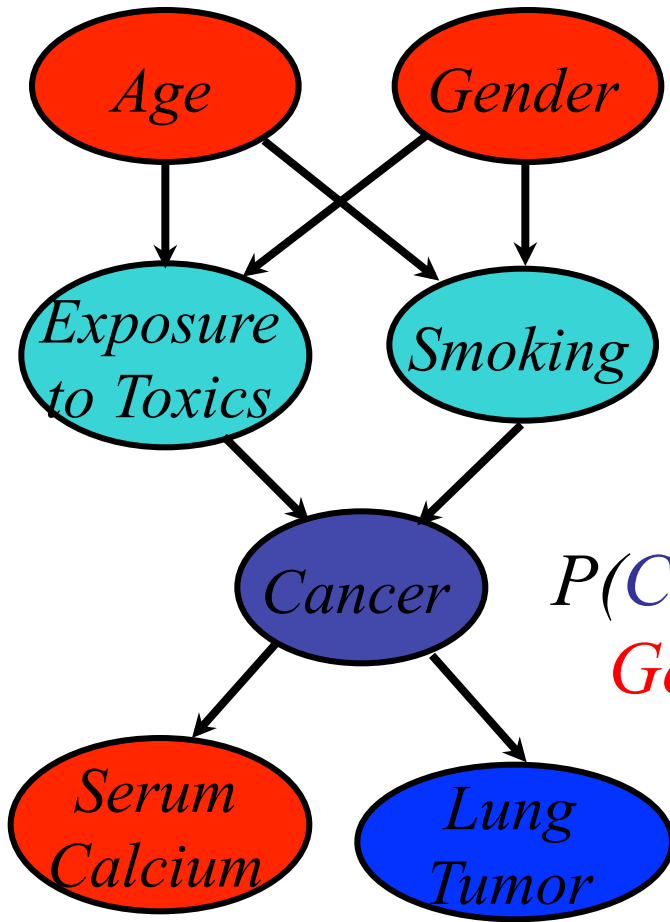
Predictive Inference



How likely are **elderly males** to get **malignant cancer**?

$$P(C=\text{malignant} \mid \text{Age} > 60, \text{Gender} = \text{male})$$

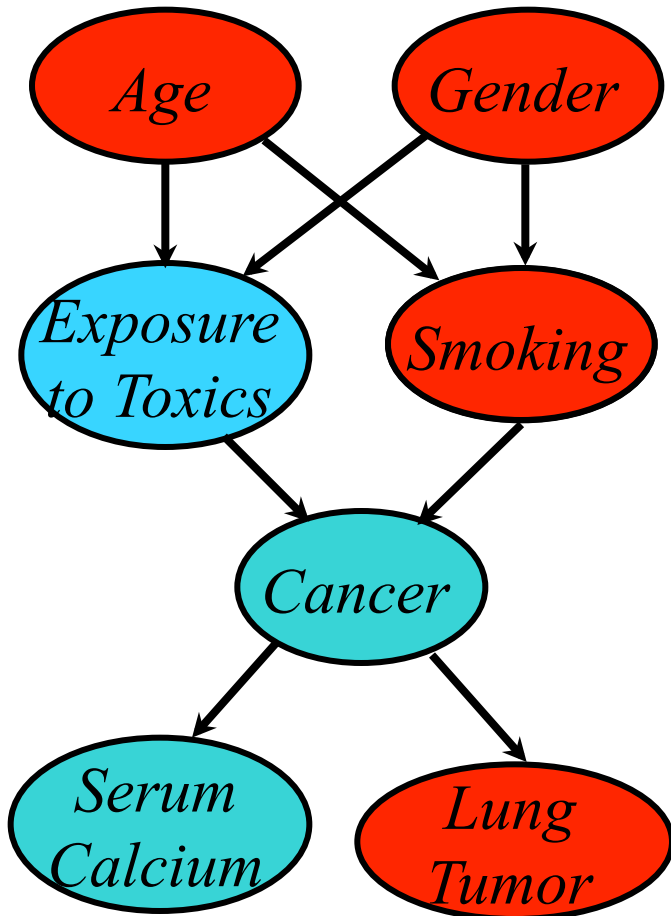
Predictive and diagnostic combined



How likely is an **elderly male** patient with high **Serum Calcium** to have malignant cancer?

$$P(C=\text{malignant} \mid \text{Age} > 60, \text{Gender} = \text{male}, \text{Serum Calcium} = \text{high})$$

Explaining away



- If we see a **lung tumor**, the probability of **heavy smoking** and of **exposure to toxics** both go up
- If we then observe **heavy smoking**, the probability of **exposure to toxics** goes back down

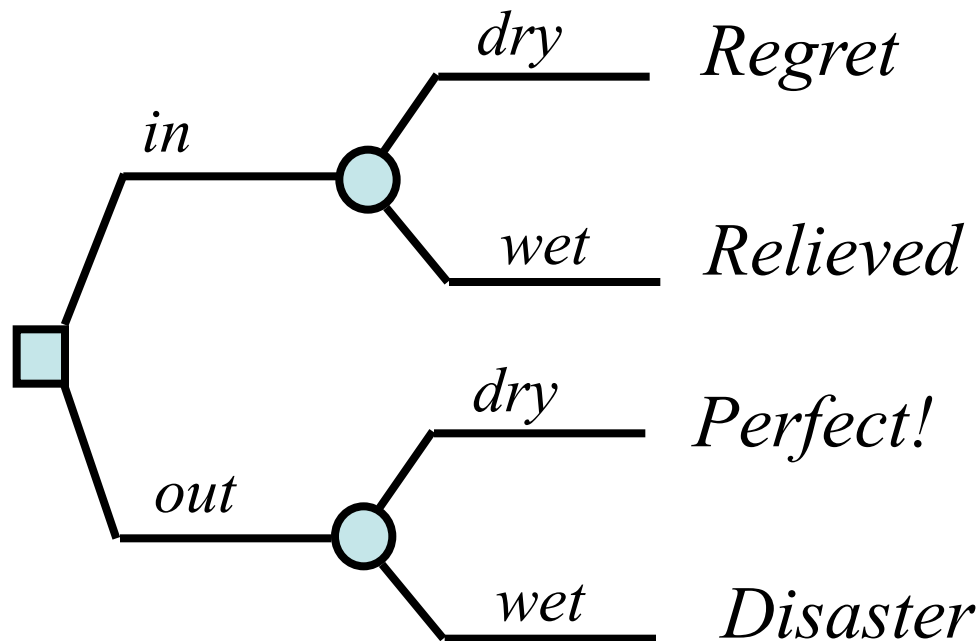
Decision making

- A decision in a medical domain might be a choice of treatment (e.g., radiation or chemotherapy)
- Decisions should be made to maximize expected utility
- View decision making in terms of
 - Beliefs/Uncertainties
 - Alternatives/Decisions
 - Objectives/Utilities

A Decision Problem



Should I have my party inside or outside?



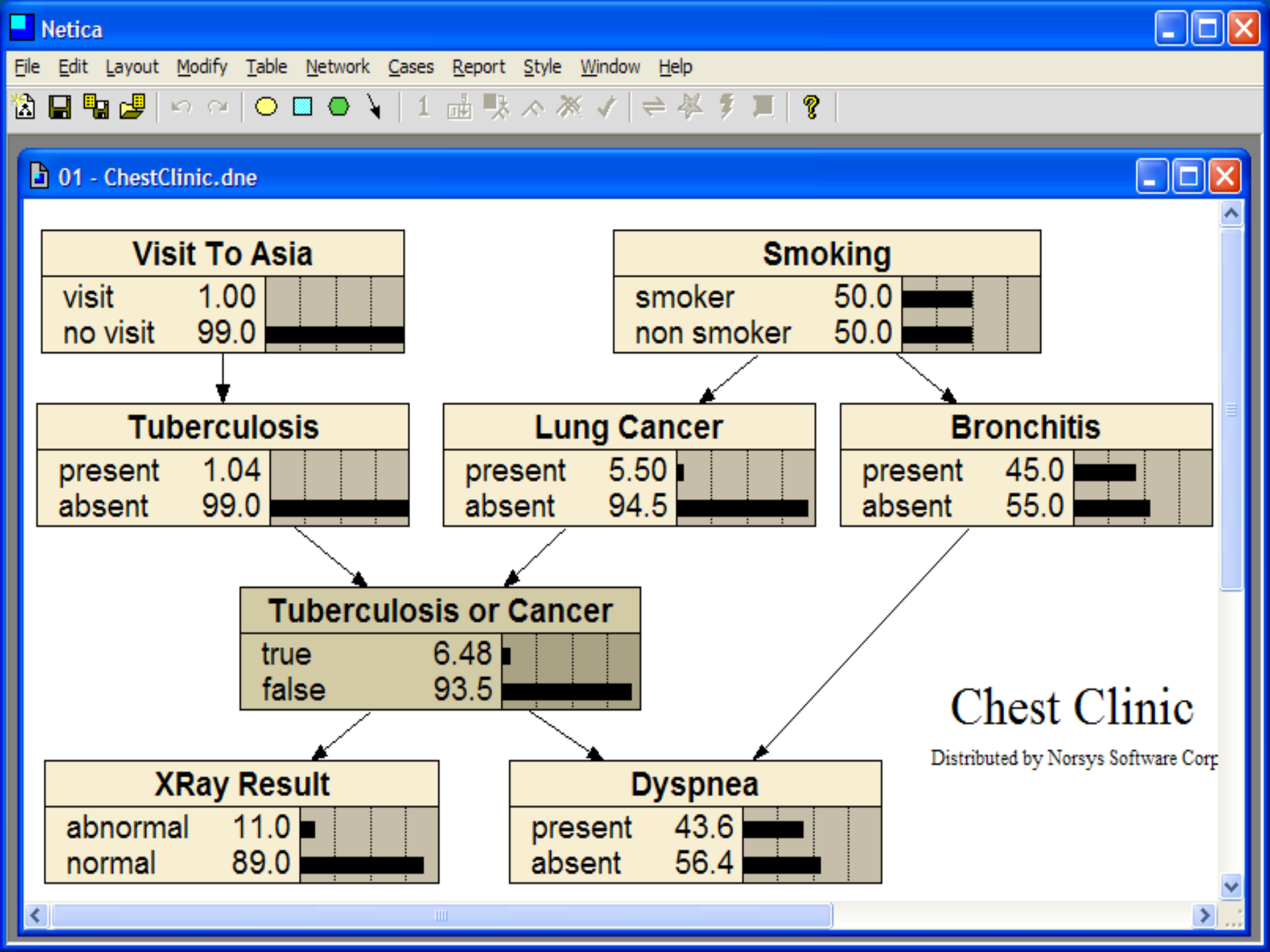
Value Function

A numerical score over all possible states of the world allows BBN to be used to make decisions

Location?	Weather?	Value
in	dry	\$50
in	wet	\$60
out	dry	\$100
out	wet	\$0

Two software tools

- [Netica](#): Windows app for working with Bayesian belief networks and influence diagrams
 - A commercial product but free for small networks
 - Includes a graphical editor, compiler, inference engine, etc.
- [Samiam](#): Java system for modeling and reasoning with Bayesian networks
 - Includes a GUI and reasoning engine



Predispositions or causes

Visit To Asia	
visit	1.00
no visit	99.0

Smoking	
smoker	50.0
non smoker	50.0

Tuberculosis	
present	1.04
absent	99.0

Lung Cancer	
present	5.50
absent	94.5

Bronchitis	
present	45.0
absent	55.0

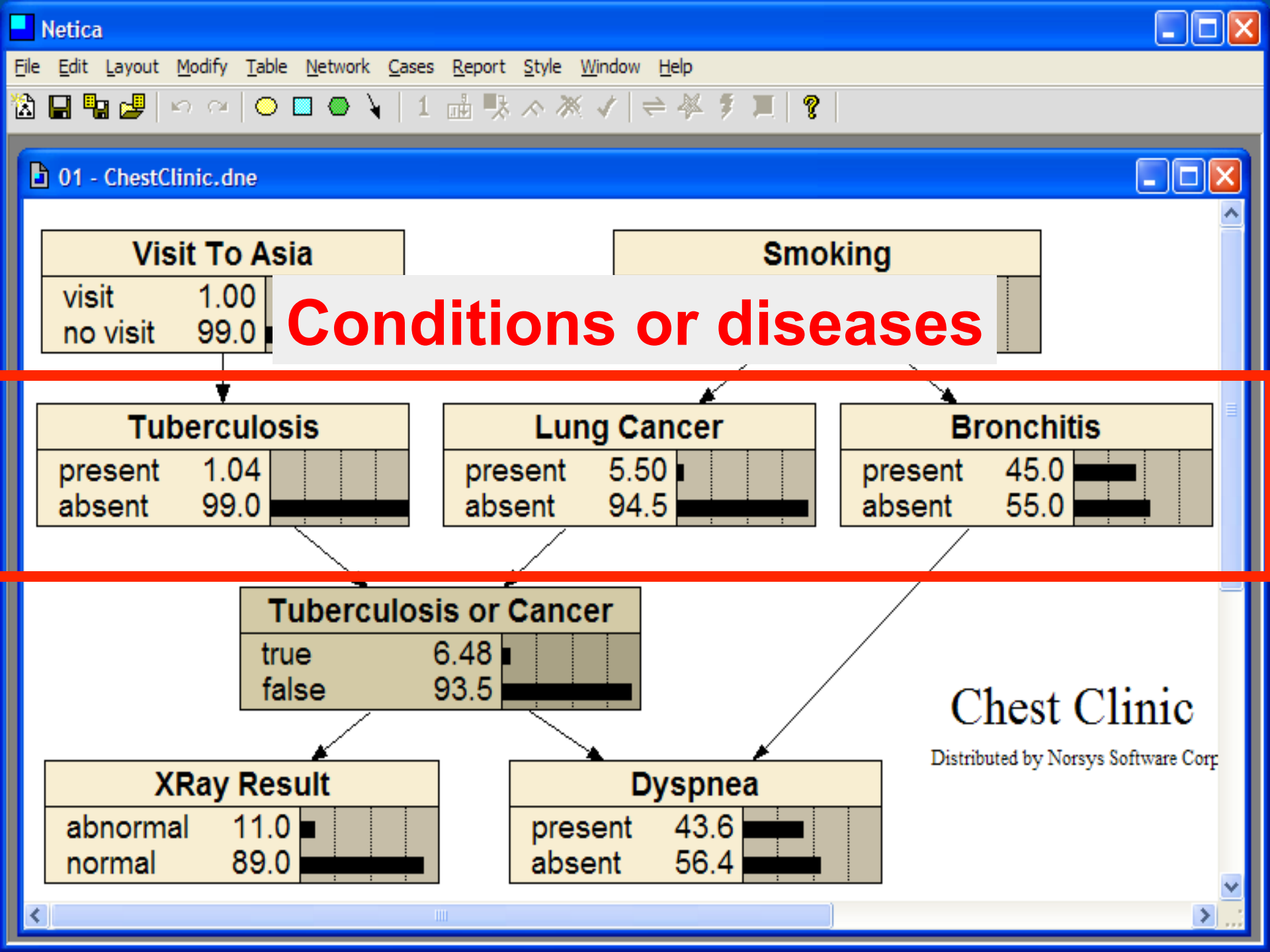
Tuberculosis or Cancer	
true	6.48
false	93.5

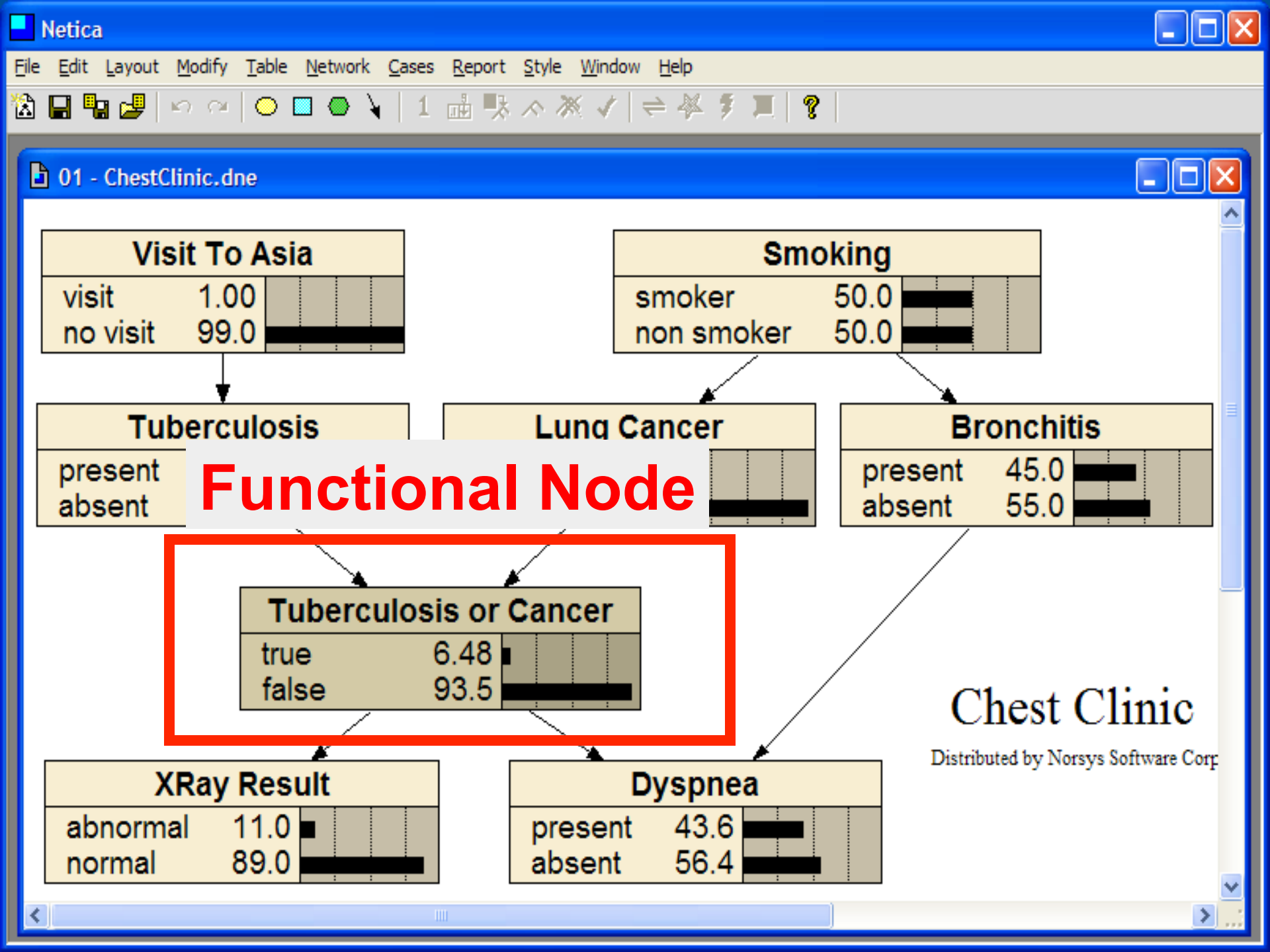
XRay Result	
abnormal	11.0
normal	89.0

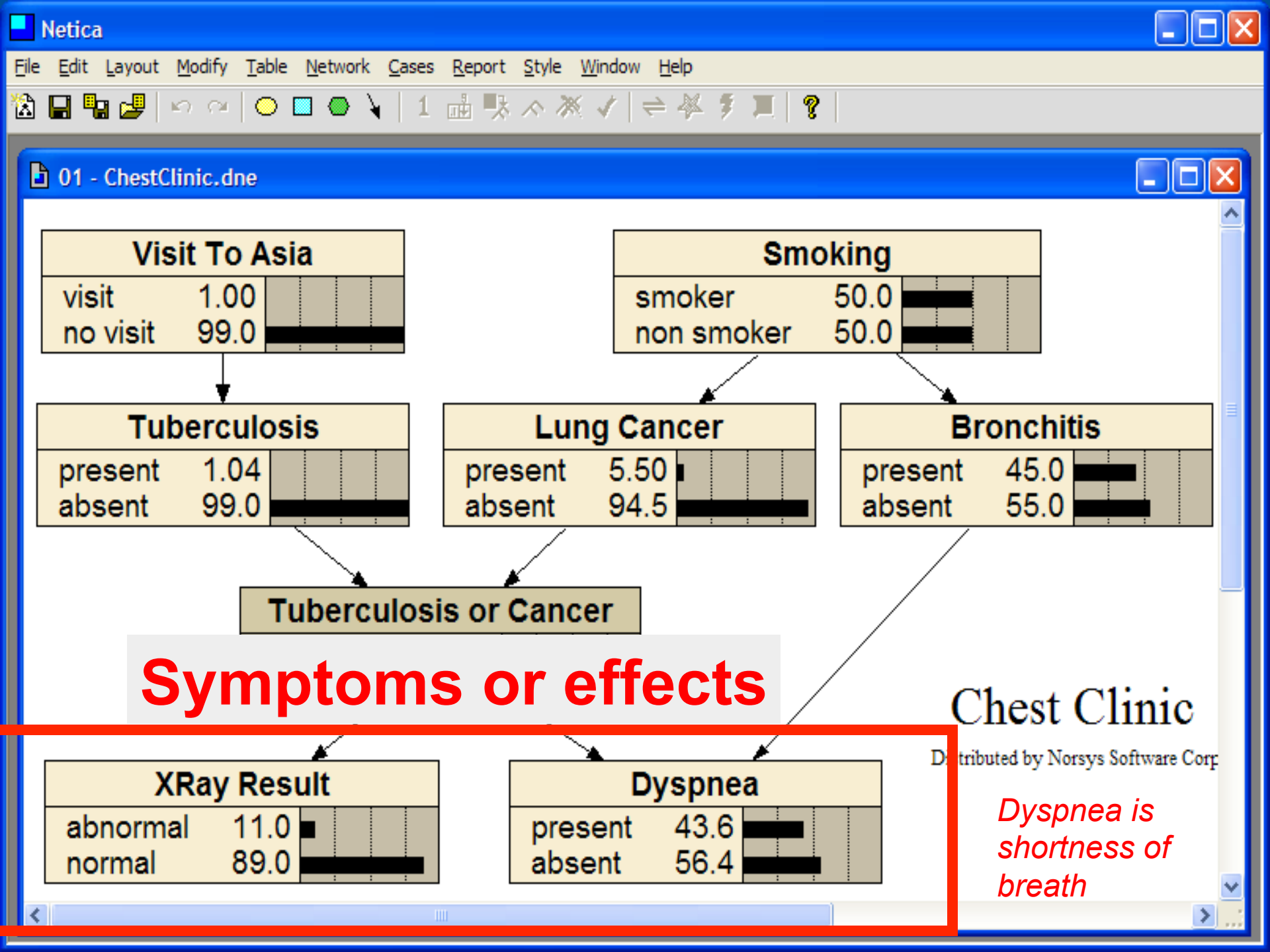
Dyspnea	
present	43.6
absent	56.4

Chest Clinic

Distributed by Norsys Software Corp







Decision Making with BBNs

- Today's weather forecast might be either sunny, cloudy or rainy
- Should you take an umbrella when you leave?
- Your decision depends only on the forecast
 - The forecast “depends on” the actual weather
- Your satisfaction depends on your decision and the weather
 - Assign a utility to each of four situations: (rain|no rain) x (umbrella, no umbrella)

Decision Making with BBNs

- Extend the BBN framework to include two new kinds of nodes: Decision and Utility
- A **Decision** node computes the expected utility of a decision given its parent(s), e.g., forecast, and a valuation
- A **Utility** node computes a utility value given its parents, e.g. a decision and weather
 - We can assign a utility to each of four situations: (rain|no rain) x (umbrella, no umbrella)
 - The value assigned to each is probably subjective



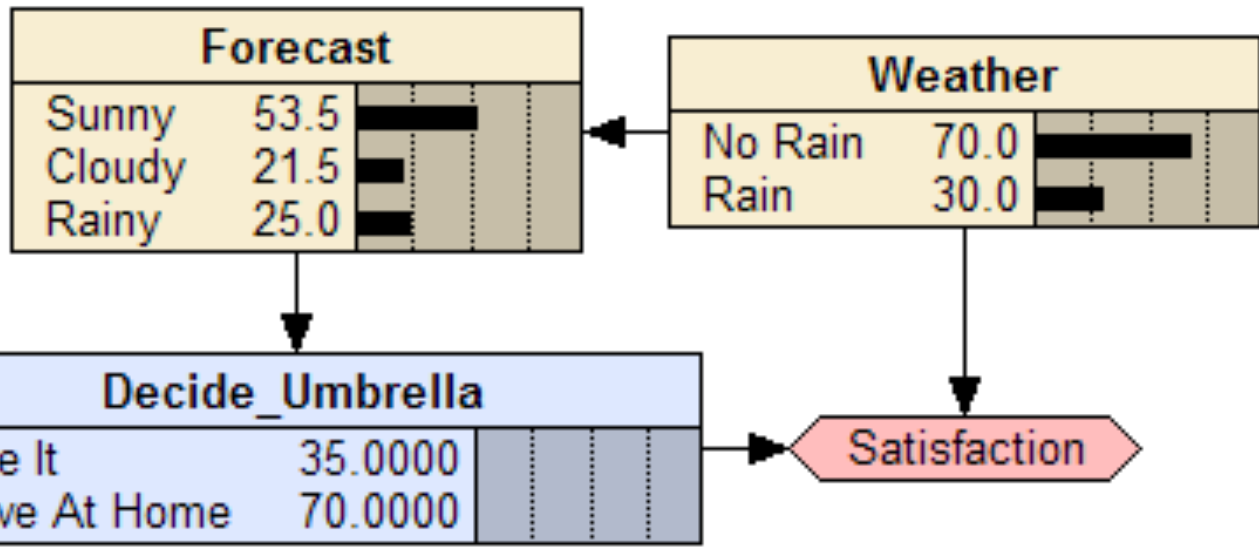
03 - Umbrella.dne

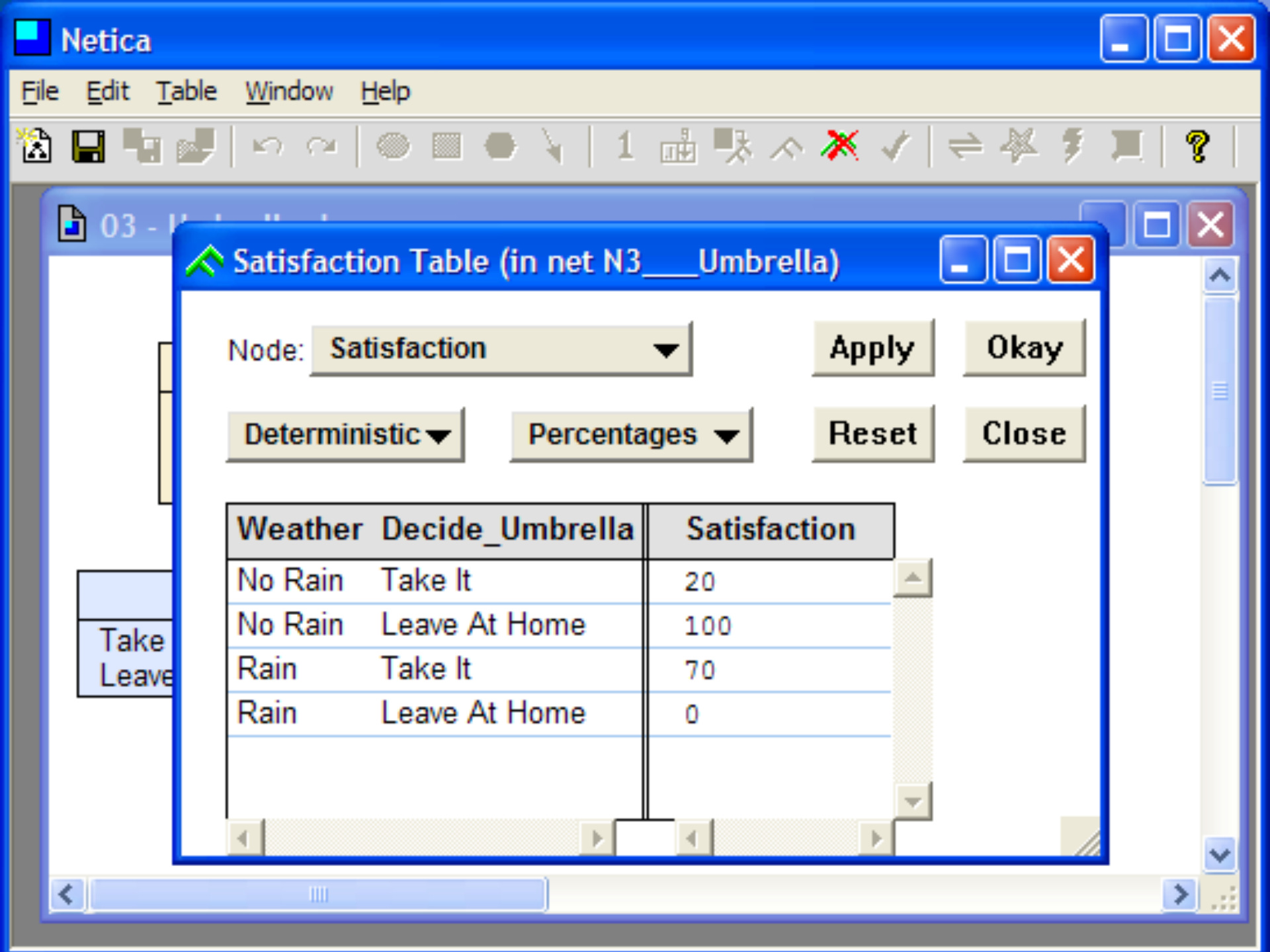
Forecast	
Sunny	53.5
Cloudy	21.5
Rainy	25.0

Weather	
No Rain	70.0
Rain	30.0

Decide_Umbrella	
Take It	35.0000
Leave At Home	70.0000

Satisfaction





03 - Umbrella

Satisfaction Table (in net N3__Umbrella)



Node: Satisfaction

Apply

Okay

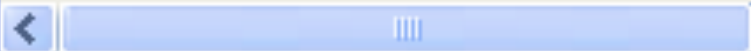
Deterministic

Percentages

Reset

Close

Weather	Decide_Umbrella	Satisfaction
No Rain	Take It	20
No Rain	Leave At Home	100
Rain	Take It	70
Rain	Leave At Home	0





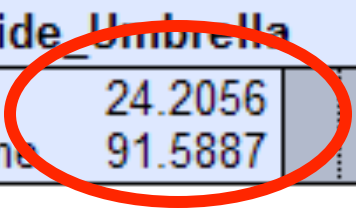
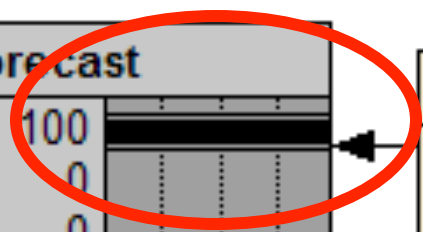
03 - Umbrella.dne

Forecast	
Sunny	100
Cloudy	0
Rainy	0

Weather	
No Rain	91.6
Rain	8.41

Decide_Umbrella	
Take It	24.2056
Leave At Home	91.5887

Satisfaction





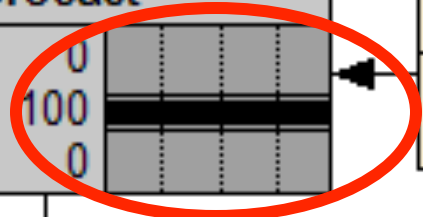
03 - Umbrella.dne

Forecast			
Sunny	0		
Cloudy	100		
Rainy	0		

Weather			
No Rain	65.1		
Rain	34.9		

Decide_Umbrella			
Take It	37.4418		
Leave At Home	65.1162		

Satisfaction





03 - Umbrella.dne

Forecast			
Sunny	0		
Cloudy	0		
Rainy	100		

Weather			
No Rain	28.0		
Rain	72.0		

Decide_Umbrella			
Take It	56.0000		
Leave At Home	28.0000		

Satisfaction

