Logical Inference 1 introduction

Chapter 9

Some material adopted from notes by
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Overview

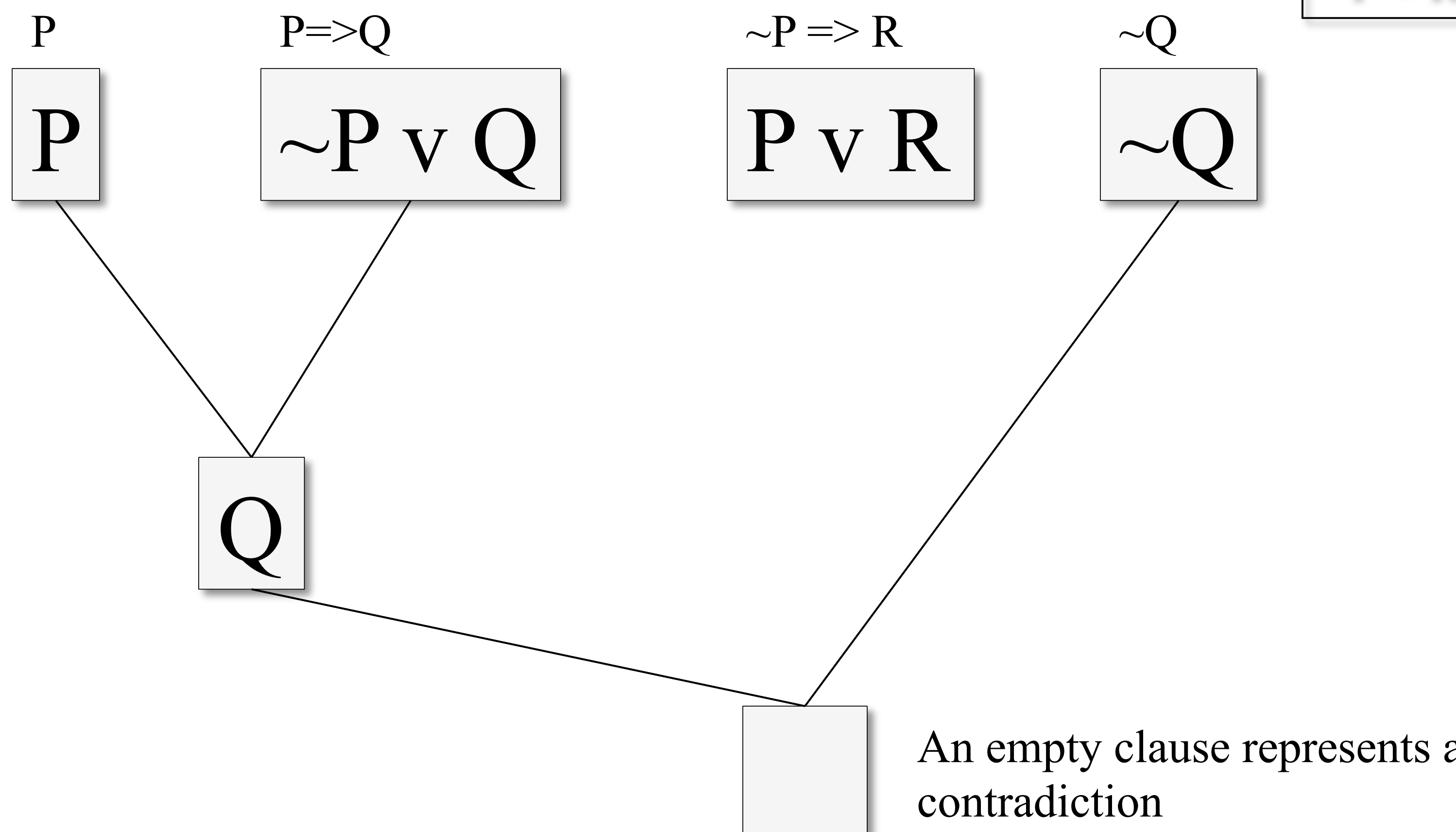
- A: Model checking for propositional logic
- Rule based reasoning in first-order logic
  - Inference rules and generalized modes ponens
  - Forward chaining
  - Backward chaining
- Resolution-based reasoning in first-order logic
  - Clausal form
  - Unification
  - Resolution as search
- Inference wrap up

From Satisfiability to Proof

- To see if a satisfiable KB entails sentence S, see if KB ∧ ¬S is satisfiable
  - If it is not, then the KB entails S
  - If it is, then the KB does not email S
  - This is a refutation proof
- Consider the KB with (P, P=>Q, ~P=>R)
  - Does the KB it entail Q? R?

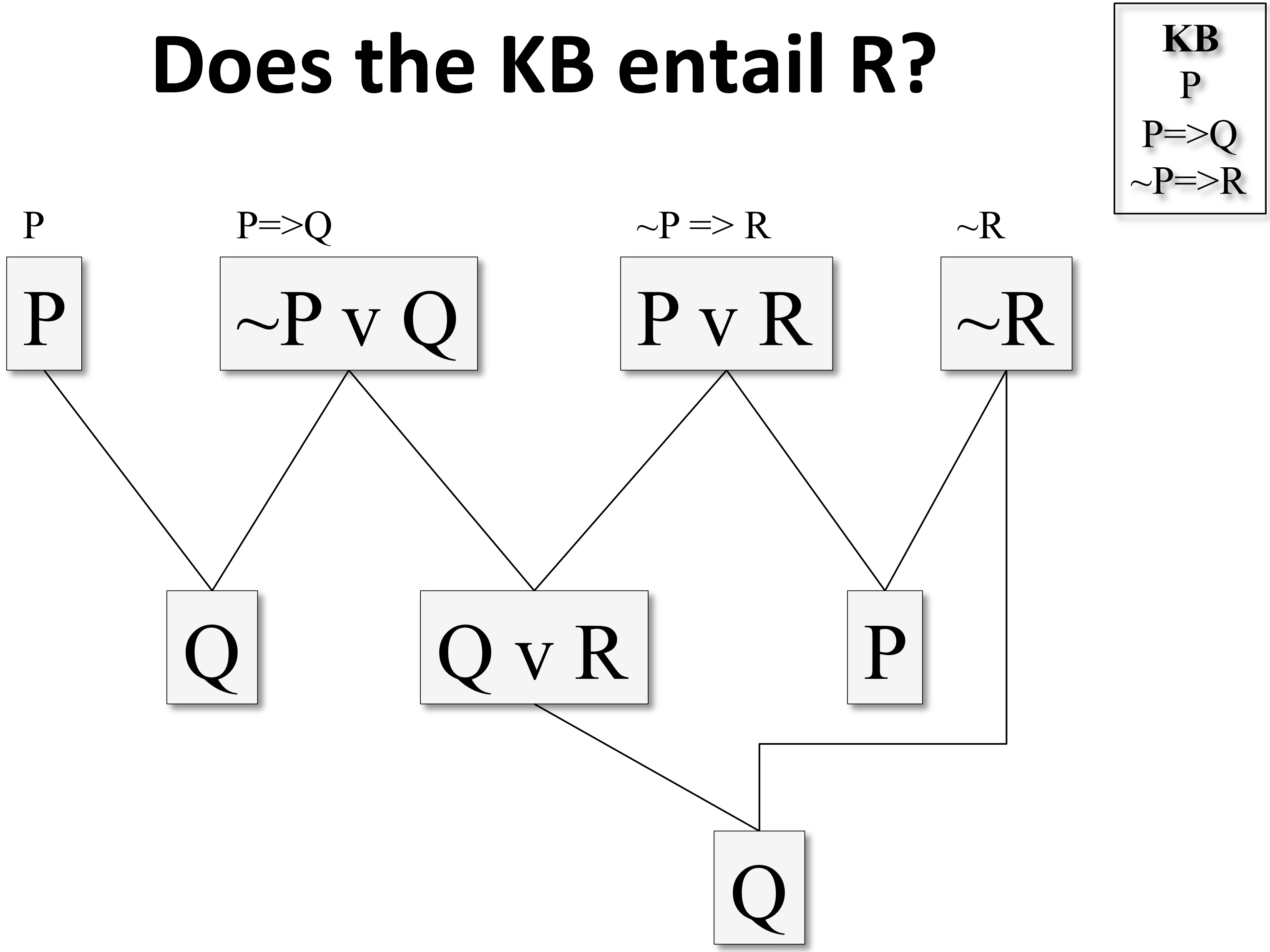
Does the KB entail Q?

KB
P
P=>Q
~P=>R



We assume that every sentence in the KB is true. Adding ~Q to the KB yields a contradiction, so ~Q must be false, so Q must be true.

Does the KB entail R?



Adding ~R to KB does not produce a contradiction after drawing all possible conclusions, so it could be False, so KB doesn't entail R.

Propositional Logic Model checking

- Given KB, does a sentence S hold?
  - All of the logic variables in S must be in the KB
- Basically generate and test:
  - Consider models M in which every sentence in the KB is TRUE
  - If $\forall M\ S$, then S is **provably true**
  - If $\forall M\ \neg S$, then S is **provably false**
  - Otherwise ($\exists M1\ S \wedge \exists M2\ \neg S$): S is **satisfiable** but neither provably true or provably false

Efficient PL model checking (1)

Davis-Putnam algorithm (DPLL) is generate-and-test model checking with several optimizations:

- *Early termination:* short-circuiting of disjunction/ conjunction
- *Pure symbol heuristic*: symbols appearing only negated or un-negated must be FALSE/TRUE respectively

  e.g., in [(A∨¬B), (¬B∨¬C), (C∨A)] A & B are pure, C impure. Make pure symbol literal true: if there's a model for S, making pure symbol true is also a model
- *Unit clause heuristic*: Symbols in a clause by itself can immediately be set to TRUE or FALSE

Using the AIMA Code

expr parses a string, and returns a logical expression

dpll_satisfiable returns a model if satisfiable else False

```
python> python
Python ...
>>> from logic import *
>>> expr('P & P==>Q & ~P==>R')
((P & (P >> Q)) & (~P >> R))

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R'))
{R: True, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~R'))
{R: False, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~Q'))
False

>>>
```

The KB entails Q but does not email R

Efficient PL model checking (2)

- WalkSAT is a local search for satisfiability: Pick a symbol to flip (toggle TRUE/FALSE), either using min-conflicts *or* choosing randomly
- …or you can use *any* local or global search algorithm!
- There are many model checking algorithms and systems
  - See for example, MiniSat
  - International SAT Competition (2003…2016)

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

The KB does not entail Q

After adding P the KB does entail Q

Retracting P removes it and the KB no longer entails Q

Reminder: Inference rules for FOL

- Inference rules for propositional logic apply to FOL as well
  - Modus Ponens, And-Introduction, And-Elimination, …
- New (sound) inference rules for use with quantifiers:
  - Universal elimination
  - Existential introduction
  - Existential elimination
  - Generalized Modus Ponens (GMP)