7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

7.3 Consider the problem of deciding whether a propositional logic sentence is true in a given model.

a. Write a recursive algorithm PL-TRUE?(s, m) that returns *true* if and only if the sentence s is true in the model m (where m assigns a truth value for every symbol in s). The algorithm should run in time linear in the size of the sentence. (Alternatively, use a version of this function from the online code repository.)

b. Give three examples of sentences that can be determined to be true or false in a *partial* model that does not specify a truth value for some of the symbols.

c. Show that the truth value (if any) of a sentence in a partial model cannot be determined efficiently in general.

d. Modify your PL-TRUE? algorithm so that it can sometimes judge truth from partial models, while retaining its recursive structure and linear run time. Give three examples of sentences whose truth in a partial model is *not* detected by your algorithm.

e. Investigate whether the modified algorithm makes TT-ENTAILS? more efficient.

7.4 Which of the following are correct?

a. $False \models True$.

b. $True \models False$.

c. $(A \land B) \models (A \Leftrightarrow B)$.

d. $A \Leftrightarrow B \models A \lor B$.

e. $A \Leftrightarrow B \models \neg A \lor B$.

f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$.

g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$.

h. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$.

i. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$.

j. $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable.

k. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable.

l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is v

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \land \neg\beta)$ is u

7.6 Prove, or find a counterexample to, each of the f

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$

b. If $\alpha \models (\beta \land \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.

c. If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.7 Consider a vocabulary with only four proposition are there for the following sentences?

a. $B \lor C$.

b. $\neg A \lor \neg B \lor \neg C \lor \neg D$.

c. $(A \Rightarrow B) \land A \land \neg B \land C \land D$.

7.8 We have defined four binary logical connectives.

a. Are there any others that might be useful?

b. How many binary connectives can there be?

c. Why are some of them not very useful?

7.9 Using a method of your choice, verify each of the

7.10 Decide whether each of the following sentences ify your decisions using truth tables or the equivalence

a. $Smoke \Rightarrow Smoke$

b. $Smoke \Rightarrow Fire$

c. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

d. $Smoke \lor Fire \lor \neg Fire$

e. $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow$

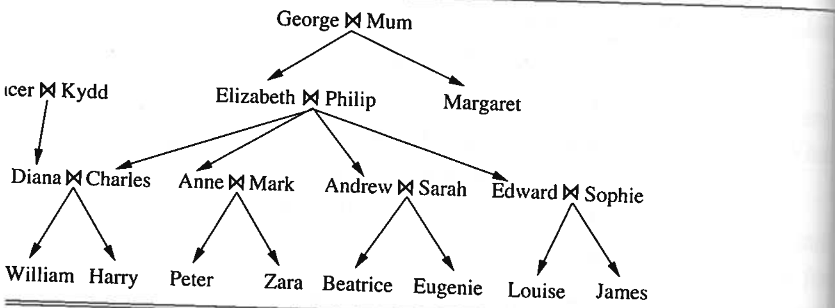
f. $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow F$

g. $Big \lor Dumb \lor (Big \Rightarrow Dumb)$

7.11 Any propositional logic sentence is logically eq sible world in which it would be false is not the case. sentence can be written in CNF.

7.12 Use resolution to prove the sentence $\neg A \land \neg B$ fr

7.13 This exercise looks into the relationship betwee



…typical family tree. The symbol "⋈" connects spouses and arrows point to

…describing the predicates *Grandchild*, *Greatgrandparent*, *Ancestor*, …ughter, *Son*, *FirstCousin*, *BrotherInLaw*, *SisterInLaw*, *Aunt*, and … proper definition of mth cousin n times removed, and write the def… logic. Now write down the basic facts depicted in the family tree in … suitable logical reasoning system, TELL it all the sentences you have …SK it who are Elizabeth's grandchildren, Diana's brothers-in-law, Zara's …nd Eugenie's ancestors.

…s wrong with the following proposed definition of the set membership

}
$\forall y \;\; x \in \{y|s\}$.

…xioms as examples, write axioms for the list domain, including all the …nd predicates mentioned in the chapter.

…s wrong with the following proposed definition of adjacent squares in

…$t([x,y],[x+1,y]) \wedge Adjacent([x,y],[x,y+1])$.

…xioms required for reasoning about the wumpus's location, using a …pus and a binary predicate $At(Wumpus, Location)$. Remember that …ous.

…cates $Parent(p,q)$ and $Female(p)$ and constants *Joan* and *Kevin*, …ings, express each of the following sentences in first-order logic. (You …on $\exists^1$ to mean "there exists exactly one.")

…er (possibly more than one, and possibly sons as well).

8.20 Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and $\times$, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.

a. Represent the property "x is an even number."

b. Represent the property "x is prime."

c. Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.

8.21 In Chapter 6, we used equality to indicate the relation between a variable and its value. For instance, we wrote $WA = red$ to mean that Western Australia is colored red. Representing this in first-order logic, we must write more verbosely $ColorOf(WA) = red$. What incorrect inference could be drawn if we wrote sentences such as $WA = red$ directly as logical assertions?

8.22 Write in first-order logic the assertion that every key and at least one of every pair of socks will eventually be lost forever, using only the following vocabulary: $Key(x)$, x is a key; $Sock(x)$, x is a sock; $Pair(x,y)$, x and y are a pair; Now, the current time; $Before(t_1,t_2)$, time t_1 comes before time t_2; $Lost(x,t)$, object x is lost at time t.

8.23 For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error!)

a. No two people have the same social security number.

$$\neg\exists\, x,y,n \;\; Person(x) \wedge Person(y) \;\Rightarrow\; [HasSS\#(x,n) \wedge HasSS\#(y,n)].$$

b. John's social security number is the same as Mary's.

$$\exists\, n \;\; HasSS\#(John,n) \wedge HasSS\#(Mary,n).$$

c. Everyone's social security number has nine digits.

$$\forall\, x,n \;\; Person(x) \;\Rightarrow\; [HasSS\#(x,n) \wedge Digits(n,9)].$$

d. Rewrite each of the above (uncorrected) sentences using a function symbol $SS\#$ instead of the predicate $HasSS\#$.

8.24 Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

a. Some students took French in spring 2001.

b. Every student who takes French passes it.

c. Only one student took Greek in spring 2001.

d. A common error among students is to suppose that, in unification, one is allowed to substitute a term for a Skolem constant instead of for a variable. For instance, they will say that the formulas $P(Sk1)$ and $P(A)$ can be unified under the substitution $\{Sk1/A\}$. Give an example where this leads to an invalid inference.

9.8 Explain how to write any given 3-SAT problem of arbitrary size using a single first-order definite clause and no more than 30 ground facts.

9.9 Suppose you are given the following axioms:

1. $0 \leq 3$.
2. $7 \leq 9$.
3. $\forall x \quad x \leq x$.
4. $\forall x \quad x \leq x + 0$.
5. $\forall x \quad x + 0 \leq x$.
6. $\forall x, y \quad x + y \leq y + x$.
7. $\forall w, x, y, z \quad w \leq y \wedge x \leq z \;\Rightarrow\; w + x \leq y + z$.
8. $\forall x, y, z \quad x \leq y \wedge y \leq z \;\Rightarrow\; x \leq z$

a. Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

b. Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Again, show only the steps that lead to success.

9.10 A popular children's riddle is "Brothers and sisters have I none, but that man's father is my father's son." Use the rules of the family domain (Section 8.3.2 on page 301) to show who that man is. You may apply any of the inference methods described in this chapter. Why do you think that this riddle is difficult?

9.11 Suppose we put into a logical knowledge base a segment of the U.S. census data listing the age, city of residence, date of birth, and mother of every person, using social security numbers as identifying constants for each person. Thus, George's age is given by $Age(443\text{-}65\text{-}1282, 56)$. Which of the following indexing schemes S1–S5 enable an efficient solution for which of the queries Q1–Q4 (assuming normal backward chaining)?

- **S1**: an index for each atom in each position.
- **S2**: an index for each first argument.
- **S3**: an index for each predicate atom.
- **S4**: an index for each *combination* of predicate and first argument.

- **Q3**: $Mother(x, y)$
- **Q4**: $Age(x, 34) \wedge ResidesIn(x, TinyTownUSA)$

9.12 One might suppose that we can avoid the proble during backward chaining by standardizing apart all of once and for all. Show that, for some sentences, this ap a sentence in which one part unifies with another.)

9.13 In this exercise, use the sentences you wrote in using a backward-chaining algorithm.

a. Draw the proof tree generated by an exhaustive query $\exists h \;\; Horse(h)$, where clauses are matched

b. What do you notice about this domain?

c. How many solutions for h actually follow from y

d. Can you think of a way to find all of them? (*Hin*

9.14 Trace the execution of the backward-chaining al it is applied to solve the crime problem (page 330). Sh the *goals* variable, and arrange them into a tree.

9.15 The following Prolog code defines a predicate variables, not constants, in Prolog.)

```
P(X, [X|Y]).
P(X, [Y|Z]) :- P(X,Z).
```

a. Show proof trees and solutions for the queries P

b. What standard list operation does P represent?

9.16 This exercise looks at sorting in Prolog.

a. Write Prolog clauses that define the predicate s list L is sorted in ascending order.

b. Write a Prolog definition for the predicate pe is a permutation of M.

c. Define `sort(L,M)` (M is a sorted version of

d. Run `sort` on longer and longer lists until you ity of your program?

e. Write a faster sorting algorithm, such as inser ing systems. Use the predicate `rewrite(X,Y)` to represent rewrite rules. For example, the earlier rewrite rule is written as `rewrite(X+0,X)`. Some terms are *primitive* and cannot be further simplified; thus, we write `primitive(0)` to say that 0 is a primitive term.

a. Write a definition of a predicate `simplify(X,Y)`, that is true when `Y` is a simplified version of `X`—that is, when no further rewrite rules apply to any subexpression of `Y`.
b. Write a collection of rules for the simplification of expressions involving arithmetic operators, and apply your simplification algorithm to some sample expressions.
c. Write a collection of rewrite rules for symbolic differentiation, and use them along with your simplification rules to differentiate and simplify expressions involving arithmetic expressions, including exponentiation.

9.18 This exercise considers the implementation of search algorithms in Prolog. Suppose that `successor(X,Y)` is true when state `Y` is a successor of state `X`; and that `goal(X)` is true when `X` is a goal state. Write a definition for `solve(X,P)`, which means that `P` is a path (list of states) beginning with `X`, ending in a goal state, and consisting of a sequence of legal steps as defined by `successor`. You will find that depth-first search is the easiest way to do this. How easy would it be to add heuristic search control?

9.19 Suppose a knowledge base contains just the following first-order Horn clauses:

$Ancestor(Mother(x), x)$
$Ancestor(x, y) \land Ancestor(y, z) \Rightarrow Ancestor(x, z)$

Consider a forward chaining algorithm that, on the jth iteration, terminates if the KB contains a sentence that unifies with the query, else adds to the KB every atomic sentence that can be inferred from the sentences already in the KB after iteration $j-1$.

a. For each of the following queries, say whether the algorithm will (1) give an answer (if so, write down that answer); or (2) terminate with no answer; or (3) never terminate.
   (i) $Ancestor(Mother(y), John)$
   (ii) $Ancestor(Mother(Mother(y)), John)$
   (iii) $Ancestor(Mother(Mother(Mother(y))), Mother(y))$
   (iv) $Ancestor(Mother(John), Mother(Mother(John)))$
b. Can a resolution algorithm prove the sentence $\neg Ancestor(John, John)$ from the original knowledge base? Explain how, or why not.
c. Suppose we add the assertion that $\neg(Mother(x) = x)$ and augment the resolution algorithm with inference rules for equality. Now what is the answer to (b)?

9.20 Let $\mathcal{L}$ be the first-order language with a single predicate $S(p, q)$, meaning "p shaves q." Assume a domain of people.

(Note: you do not need ...

9.21 How can resolution be used to show tha

9.22 Construct an example of two clauses t ways giving two different outcomes.

9.23 From "Horses are animals," it follows animal." Demonstrate that this inference is val

a. Translate the premise and the conclusion predicates: $HeadOf(h, x)$ (meaning "h
b. Negate the conclusion, and convert the junctive normal form.
c. Use resolution to show that the conclusi

9.24 Here are two sentences in the language

(A) $\forall x \; \exists y \; (x \geq y)$
(B) $\exists y \; \forall x \; (x \geq y)$

a. Assume that the variables range over al "$\geq$" predicate means "is greater than c (A) and (B) into English.
b. Is (A) true under this interpretation?
c. Is (B) true under this interpretation?
d. Does (A) logically entail (B)?
e. Does (B) logically entail (A)?
f. Using resolution, try to prove that (A) (B) does not logically entail (A); cont proceed (if it does break down). Show If the proof fails, explain exactly whe
g. Now try to prove that (B) follows fror

9.25 Resolution can produce nonconstruc to introduce special mechanisms to extract arise with knowledge bases containing onl

9.26 We said in this chapter that resolut quences of a set of sentences. Can any alg

ɔoks at the recursive application of rewrite rules, using logic program-
ɔr **demodulator** in OTTER terminology) is an equation with a specified
e, the rewrite rule $x + 0 \rightarrow x$ suggests replacing any expression that
expression x. Rewrite rules are a key component of equational reason-
redicate `rewrite(X,Y)` to represent rewrite rules. For example, the
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$ther(y), John)$

$ther(Mother(y)), John)$

$ther(Mother(Mother(y))), Mother(y))$

$ther(John), Mother(Mother(John)))$

ɔrithm prove the sentence $\neg Ancestor(John, John)$ from the orig-
? Explain how, or why not.

assertion that $\neg(Mother(x) = x)$ and augment the resolution al-

a. Consider the sentence "There exists a person P who shaves every one who does not shave themselves, and only people that do not shave themselves." Express this in $\mathcal{L}$.

b. Convert the sentence in (a) to clausal form.

c. Construct a resolution proof to show that the clauses in (b) are inherently inconsistent. (Note: you do not need any additional axioms.)

9.21 How can resolution be used to show that a sentence is valid? Unsatisfiable?

9.22 Construct an example of two clauses that can be resolved together in two different ways giving two different outcomes.

9.23 From "Horses are animals," it follows that "The head of a horse is the head of an animal." Demonstrate that this inference is valid by carrying out the following steps:

a. Translate the premise and the conclusion into the language of first-order logic. Use three predicates: $HeadOf(h, x)$ (meaning "h is the head of x"), $Horse(x)$, and $Animal(x)$.

b. Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.

c. Use resolution to show that the conclusion follows from the premise.

9.24 Here are two sentences in the language of first-order logic:

(A) $\forall x\ \exists y\ (x \geq y)$

(B) $\exists y\ \forall x\ (x \geq y)$

a. Assume that the variables range over all the natural numbers $0, 1, 2, \ldots, \infty$ and that the "$\geq$" predicate means "is greater than or equal to." Under this interpretation, translate (A) and (B) into English.

b. Is (A) true under this interpretation?

c. Is (B) true under this interpretation?

d. Does (A) logically entail (B)?

e. Does (B) logically entail (A)?

f. Using resolution, try to prove that (A) follows from (B). Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how, and why it breaks down.

g. Now try to prove that (B) follows from (A).

9.25 Resolution can produce nonconstructive proofs for queries with variables, so we had