CMSC 341 Data Structures
Red-Black Tree Review

October 26, 2010

These questions will help test your understanding of the Red-Black tree material discussed in class and in the text. These questions are only a study guide. Questions found here may be on your exam, although perhaps in a different format. Questions NOT found here may also be on your exam. The rotation diagrams for red-black trees ***may*** be provided with your exam. Check with your instructor.eps

1. Define ***Red-Black tree***.
2. Define the ***black height*** of a node, x.
3. What is the "big-Oh" performance (in terms of the number of nodes in the tree) for the operations **find, insert**, and **remove** for a red-black tree in the best, worst and average cases?
4. What property of red-black trees is most significant in explaining their "big-Oh" behvior for the operations **find, insert**, and **remove**?
5. Prove that in any red-black tree with root x, there are at least $n = 2^{bh(x)}-1$ internal nodes where $bh(x)$ is the black-height of x.
6. Prove that in any red-black tree, at least half the nodes on any path from the root to a leaf must be black.
7. Prove that in any red-black tree, no path from any node, N, to a leaf is more than twice as long as any other path from N to any other leaf.
8. Prove that if a black node has just one child, that child must be red.

9. Show the tree that results from inserting the values **2, 1, 4, 5, 9, 3, 6, 7** into an initially empty red-black tree. Show the tree after each insertion. Do this using both bottom-up and top-down insertion.

10. Given the following Red-Black Tree, show the tree that results after deleting the node with value **40** using bottom-up deletion.