

# CMSC 341 Data Structures

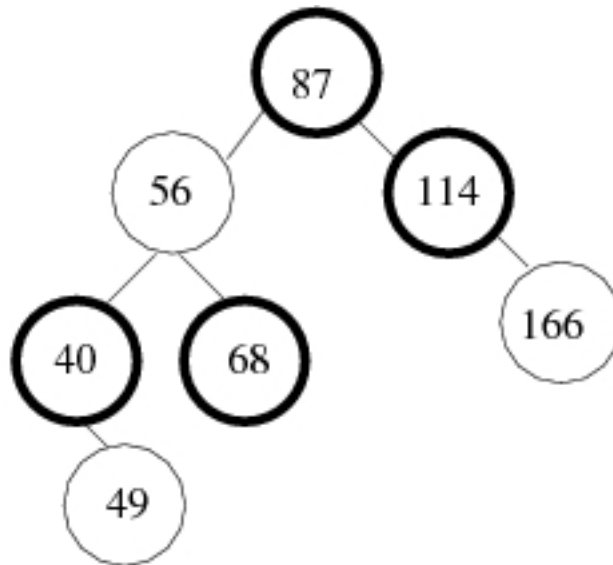
## Red-Black Tree Review

October 26, 2010

These questions will help test your understanding of the Red-Black tree material discussed in class and in the text. These questions are only a study guide. Questions found here may be on your exam, although perhaps in a different format. Questions NOT found here may also be on your exam. The rotation diagrams for red-black trees *may* be provided with your exam. Check with your instructor.eps

1. Define *Red-Black tree*.
2. Define the *black height* of a node,  $x$ .
3. What is the “big-Oh” performance (in terms of the number of nodes in the tree) for the operations **find**, **insert**, and **remove** for a red-black tree in the best, worst and average cases?
4. What property of red-black trees is most significant in explaining their “big-Oh” behavior for the operations **find**, **insert**, and **remove**?
5. Prove that in any red-black tree with root  $x$ , there are at least  $n = 2^{bh(x)} - 1$  internal nodes where  $bh(x)$  is the black-height of  $x$ .
6. Prove that in any red-black tree, at least half the nodes on any path from the root to a leaf must be black.
7. Prove that in any red-black tree, no path from any node,  $N$ , to a leaf is more than twice as long as any other path from  $N$  to any other leaf.
8. Prove that if a black node has just one child, that child must be red.

9. Show the tree that results from inserting the values **2, 1, 4, 5, 9, 3, 6, 7** into an initially empty red-black tree. Show the tree after each insertion. Do this using both bottom-up and top-down insertion.
10. Given the following Red-Black Tree, show the tree that results after deleting the node with value **40** using bottom-up deletion.



Represents a BLACK node



Represents a RED node