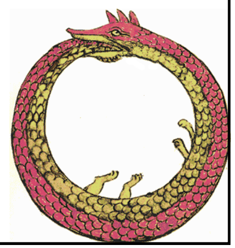
Tail Recursion



Problems with Recursion

- Recursion is generally favored over iteration in Scheme and many other languages
  - It's elegant, minimal, can be implemented with regular functions and easier to analyze formally
  - Some languages don't have iteration (Prolog)
- It can also be less efficient
  - more functional calls and stack operations (context saving and restoration)
- Running out of stack space leads to failure deep recursion

Tail recursion is iteration

- [Tail recursion](#) is a pattern of use that can be compiled or interpreted as iteration, avoiding the inefficiencies
- A tail recursive function is one where every recursive call is the last thing done by the function before returning and thus produces the function's value
- More generally, we identify some proceedure calls as [tail calls](#)

Tail Call

A *tail call* is a procedure call inside another procedure that returns a value which is then immediately returned by the calling procedure

```
def foo(data):
    bar1(data)
    return bar2(data)
```

```
def foo(data):
    if test(data):
        return bar2(data)
    else:
        return bar3(data)
```

A tail call need not come at the textual end of the procedure, but at one of its logical ends

Tail call optimization

- When a function is called, we must remember the place it was called from so we can return to it with the result when the call is complete
- This is typically stored on the call stack
- There is no need to do this for tail calls
- Instead, we leave the stack alone, so the newly called function will return its result directly to the original caller

Scheme's top level loop

- Consider a simplified version of the REPL

```
(define (repl)
  (printf "> ")
  (print (eval (read)))
  (repl))
```

- This is an easy case: with no parameters there is not much context

Scheme's top level loop 2

- Consider a fancier REPL

```
(define (repl) (repl1 0))

(define (repl1 n)
  (printf "~s> " n)
  (print (eval (read)))
  (repl1 (add1 n)))
```

- This is only slightly harder: just modify the local variable n and start at the top

Scheme's top level loop 3

- There might be more than one tail recursive call

```
(define (repl1 n)
  (printf "~s> " n)
  (print (eval (read)))
  (if (= n 9)
    (repl1 0)
    (repl1 (add1 n))))
```

- What's important is that there's nothing more to do in the function after the recursive calls

Two skills

- Distinguishing a trail recursive call from a non tail recursive one
- Being able to rewrite a function to eliminate its non-tail recursive calls

Simple Recursive Factorial

```
(define (fact1 n)
 ;; naive recursive factorial
 (if (< n 1)
   1
   (* n (fact1 (sub1 n)) )))
```

Is this a tail call?

No. It must be called and its value returned before the multiplication can be done

Tail recursive factorial

```
(define (fact2 n)
  ; rewrite to just call the tail-recursive
  ; factorial with the appropriate initial values
  (fact2.1 n 1))

(define (fact2.1 n accumulator)
   ; tail recursive factorial calls itself
   ; as last thing to be done
  (if (< n 1)
     accumulator
     (fact2.1 (sub1 n) (* accumulator n)) ))
```

Is this a tail call?

Yes. Fact2.1's args are evaluated before it's called.

Trace shows what's going on

```
> (requireracket/trace)
> (load "fact.ss")
> (trace fact1)
> (fact1 6)
```

```
|(fact1 6)
| (fact1 5)
| |(fact1 4)
| | (fact1 3)
| | |(fact1 2)
| | | (fact1 1)
| | | |(fact1 0)
| | | |1
| | | 1
| | |2
| | 6
| |24
| 120
|720
720
```

fact2

```
> (trace fact2 fact2.1)
> (fact2 6)
|(fact2 6)
|(fact2.1 6 1)
|(fact2.1 5 6)
|(fact2.1 4 30)
|(fact2.1 3 120)
|(fact2.1 2 360)
|(fact2.1 1 720)
|(fact2.1 0 720)
|720
720
```

- Interpreter & compiler note the last expression to be evaled & returned in fact2.1 is a recursive call
- Instead of pushing state on the sack, it reassigns the local variables and jumps to beginning of the procedure
- Thus, the recursion is automatically transformed into iteration

Reverse a list

- This version works, but has two problems

```
(define (rev1 list)
  ; returns the reverse a list
  (if (null? list)
     empty
     (append (rev1 (rest list))  (list (first list)))))
```

- It is not tail recursive
- It creates needless temporary lists

A better reverse

```
(define  (rev2 list) (rev2.1 list empty))

(define (rev2.1 list reversed)
  (if (null? list)
     reversed
     (rev2.1 (rest list)
             (cons (first list) reversed))))
```

rev1 and rev2

```
> (load "reverse.ss")
> (trace rev1 rev2 rev2.1)
> (rev1 '(a b c))
|(rev1 (a b c))
| (rev1 (b c))
| |(rev1 (c))
| | (rev1 ())
| | ()
| |(c)
| (c b)
|(c b a)
(c b a)
```

```
> (rev2 '(a b c))
|(rev2 (a b c))
|(rev2.1 (a b c) ())
|(rev2.1 (b c) (a))
|(rev2.1 (c) (b a))
|(rev2.1 () (c b a))
|(c b a)
(c b a)
>
```

The other problem

- Append copies the top level list structure of it's first argument.
- *(append '(1 2 3) '(4 5 6))* creates a copy of the list (1 2 3) and changes the last cdr pointer to point to the list (4 5 6)
- In reverse, each time we add a new element to the end of the list, we are (re-)copying the list.

Append (two args only)

```
(define (append list1 list2)
  (if (null? list1)
     list2
     (cons (first list1)
           (append (rest list1) list2))))
```

Why does this matter?

- The repeated rebuilding of the reversed list is needless work
- It uses up memory and adds to the cost of garbage collection (GC)
- GC adds a significant overhead to the cost of any system that uses it
- Experienced programmers avoid algorithms that needlessly consume memory that must be garbage collected

Fibonacci

- Another classic recursive function is computing the nth number in the fibonacci series

```
(define (fib n)
  (if (< n 2)
    n
    (+ (fib (- n 1))
       (fib (- n 2)))))
```

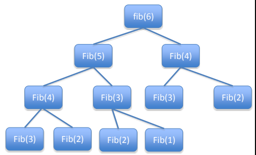
Are the tail calls?

- But its grossly inefficient
  - Run time for fib(n) ≅ $O(2^n)$
  - (fib 100) can not be computed this way

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}	F_{20}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181	6765

This has two problems

- That recursive calls are not tail recursive is the least of its problems
- It also needlessly recomputes many values



Trace of (fib 6)

```
> (fib 6)
>(fib 6)
> (fib 5)
> >(fib 4)
> > (fib 3)
> > >(fib 2)
> > > (fib 1)
< < < 1
> > > (fib 0)
< < < 0
< < <1
> > >(fib 1)
< < <1
< < 2
> > (fib 2)
> > >(fib 1)
< < <1
> > >(fib 0)
< < <0
< < 1
< <3
> >(fib 3)
> > (fib 2)
> > >(fib 1)
```

```
< < <1
> > >(fib 0)
< < <0
< < 1
> > (fib 1)
< < 1
< <2
< 5
> (fib 4)
> >(fib 3)
> > (fib 2)
> > >(fib 1)
< < <1
> > >(fib 0)
< < <0
< < 1
> > (fib 1)
< < 1
< <2
> >(fib 2)
> > (fib 1)
< < 1
> > (fib 0)
< < 0
< <1
< 3
<8
8
>
```

Tail-recursive version of Fib

Here's a tail-recursive version that runs in 0(n)

```
(define (fib2 n)
  (cond ((= n 0) 0)
        ((= n 1) 1)
        (#t (fib-tr n 2 0 1))))
(define (fib-tr target n f2 f1 )
  (if (= n target)
      (+ f2 f1)
      (fib-tr target (+ n 1) f1  (+ f1 f2))))
```

We pass four args: *n* is the current index, *target* is the index of the number we want, *f2* and *f1* are the two previous fib numbers

Trace of (fib2 10)

```
> (fib2 10)
>(fib2 10)
>(fib-tr 10 2 0 1)
>(fib-tr 10 3 1 1)
>(fib-tr 10 4 1 2)
>(fib-tr 10 5 2 3)
>(fib-tr 10 6 3 5)
>(fib-tr 10 7 5 8)
>(fib-tr 10 8 8 13)
>(fib-tr 10 9 13 21)
>(fib-tr 10 10 21 34)
<55
55
```

10 is the target, 5 is the current index fib(3)=2 and fib(4)=3

Stop when current index equals target and return sum of last two args

Compare to an iterative version

- The tail recursive version passes the "loop variables" as arguments to the recursive calls
- It's just a way to do iteration using recursive functions without the need for special iteration operators

```
def fib(n):
   if n < 3:
      return 1
   else:
      f2 = f1 = 1
      x = 3
      while x<n:
         f1, f2 = f1 + f2, f1
       return f1 + f2
```

No tail call elimination in many PLs

- Many languages don't optimize tail calls, including C, Java and Python
- Recursion depth is constrained by the space allocated for the call stack
- This is a design decision that might be justified by the worse is better principle
- See Guido van Rossum's comments on TRE

Python example

```
> def dive(n=1):
...    print n,
...    dive(n+1)
...
>>> dive()
1 2 3 4 5 6 7 8 9 10 ... 998 999
Traceback (most recent call last):
 File "<stdin>", line 1, in <module>
 File "<stdin>", line 3, in dive
 ... 994 more lines ...
 File "<stdin>", line 3, in dive
 File "<stdin>", line 3, in dive
 File "<stdin>", line 3, in dive
RuntimeError: maximum recursion depth exceeded
>>>
```

Conclusion

- Recursion is an elegant and powerful control mechanism
- We don't need to use iteration
- We can eliminate any inefficiency if we
  - Recognize and optimize tail-recursive calls, turning recursion into iteration
- Some languages (e.g., Python) choose not to do this, and advocate using iteration when appropriate

But side-effect free programming remains easier to analyze and parallelize