

Curry



A Tasty dish?



Haskell Curry!

Curried Functions

- Currying is a functional programming technique that takes a function of N arguments and produces a related one where some of the arguments are fixed
- In Scheme
 - (define add1 (curry + 1))
 - (define double (curry * 2))

A tasty dish?

- Currying was named after the Mathematical logician [Haskell Curry](#) (1900-1982)
- Curry worked on [combinatory logic](#) ...
- A technique that eliminates the need for variables in [mathematical logic](#) ...
- and hence computer programming!
 - At least in theory
- The functional programming language [Haskell](#) is also named in honor of Haskell Curry

Functions in Haskell



- In Haskell we can define g as a function that takes two arguments of types a and b and returns a value of type c like this:
 - $g :: (a, b) \rightarrow c$
- We can let f be the curried form of g by
 - $f = \text{curry } g$
- The function f now has the signature
 - $f :: a \rightarrow b \rightarrow c$
- f takes an arg of type a & returns a function that takes an arg of type b & returns a value of type c

Functions in Haskell

- All functions in Haskell are curried, i.e., *all Haskell functions take just single arguments.*
- This is mostly hidden in notation, and is not apparent to a new Haskeller
- Let's take the function $\text{div} :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$ which performs integer division
- The expression $\text{div } 11 \ 2$ evaluates to 5
- But it's a two-part process
 - $\text{div } 11$ is eval'd & returns a function of type $\text{Int} \rightarrow \text{Int}$
 - That function is applied to the value 2, yielding 5

Currying in Scheme

- Scheme has an explicit built in function, *curry*, that takes a function and some of its arguments and returns a curried function
- For example:
 - (define add1 (curry + 1))
 - (define double (curry * 2))
- We could define this easily as:


```
(define (curry fun . args)
  (lambda x (apply fun (append args x))))
```

Note on lambda syntax

- (lambda X (foo X)) is a way to define a lambda expression that takes any number of arguments
- In this case X is bound to the list of the argument values, e.g.:


```
> (define f (lambda x (print x)))
> f
#<procedure:f>
> (f 1 2 3 4 5)
(1 2 3 4 5)
>
```

Simple example (a)

- Compare two lists of numbers pair wise:


```
(apply and (map < '(0 1 2 3) '(5 6 7 8)))
```
- Note that (map < '(0 1 2 3) '(5 6 7 8)) evaluates to the list (#t #t #t #t)
- Applying and to this produces the answer, #t

Simple example (b)

- Is every number in a list positive?


```
(apply and (map < 0 '(5 6 7 8)))
```
- This is a nice idea, but will not work


```
map: expects type <proper list> as 2nd argument, given: 0; other
arguments were: #<procedure:<> (5 6 7 8)

=== context ===
/Applications/PLT/collects/scheme/private/misc.ss:74:7
```
- Map takes a function and lists for each of its arguments

Simple example (c)

- Is every number in a list positive?
- Use (lambda (x) (< 0 x)) as the function


```
(apply and (map (lambda (x) (< 0 x)) '(5 6 7 8)))
```
- This works nicely and gives the right answer
- What we did was to use a general purpose, two-argument comparison function (?<?) to make a narrower one-argument one (0<?)

Simple example (d)

- Here's where curry helps


```
(curry < 0) ≈ (lambda (x) (< 0 x))
```
- So this does what we want


```
(apply and (map (curry < 0) '(5 6 7 8)))
```

– Currying < with 0 actually produces equivalent of:

```
(lambda x (apply < (append '(0) x)))
```

– So (curry < 0) takes one or more args, e.g.

```
((curry < 0) 10 20 30) => #t
((curry < 0) 10 20 5) => #f
```

[But '< taking more than 2 args makes example a toy☺]

A real world example

- I wanted to adapt a Lisp example by Google's [Peter Norvig](#) of a simple program that generates random sentences from a context free grammar
- It was written to take the grammar and start symbol as global variables ☹
- I wanted to make this a parameter, but it made the code more complex ☹☹
- Scheme's curry helped solve this!

```
#lang scheme

;; This is a simple ...

(define grammar
  '((S -> (NP VP) (NP VP) (NP VP) (NP VP) (S CONJ S))
    (NP -> (ARTICLE ADJS? NOUN PP?))
    (VP -> (VERB NP) (VERB NP) (VERB NP) VERB)
    (ARTICLE -> the the the a a a one every)
    (NOUN -> man ball woman table penguin student book
      dog worm computer robot )

    ...
    (PP -> (PREP NP))
    (PP? -> () () () () PP)
  ))
```

[cfg1.ss](#)

```
scheme> scheme
Welcome to MzScheme v4.2.4 ...

> (require "cfg1.ss")
> (generate 'S)
(a woman took every mysterious ball)
> (generate 'S)
(a blue man liked the worm over a mysterious woman)
> (generate 'S)
(the large computer liked the dog in every mysterious student in the
mysterious dog)
> (generate 'NP)
(a worm under every mysterious blue penguin)
> (generate 'NP)
(the book with a large large dog)
```

[cfg1.ss](#)
session

```
#lang scheme

;; This is a simple ...

(define grammar
  '((S -> (NP VP) (NP VP) (NP VP) (NP VP) (S CONJ S))
    (NP -> (ARTICLE ADJS? NOUN PP?))
    (VP -> (VERB NP) (VERB NP) (VERB NP) VERB)
    (ARTICLE -> the the the a a a one every)
    (NOUN -> man ball woman table penguin student book
      dog worm computer robot )

    ...
    (PP -> (PREP NP))
    (PP? -> () () () () PP)
  ))
```

Five possible rewrites for a S:
80% of the time it => NP VP and
20% of the time it is a conjoined
sentence, S CONJ S

Terminal symbols
(e.g. the, a)
are recognized by
virtue of not
heading a
grammar rule.

() is like ε in a rule, so
80% of the time a PP?
produces nothing and
20% a PP.

```
(define (generate phrase)
  ;; generate a random sentence or phrase
  (cond ((list? phrase)
        (apply append (map generate phrase)))
        ((non-terminal? phrase)
         (generate (random-element (rewrites phrase))))
        (else (list phrase))))

(define (non-terminal? x)
  ;; True iff x is a non-terminal in grammar
  (assoc x grammar))

(define (rewrites non-terminal)
  ;; Return a list of the possible rewrites for non-terminal in grammar
  (rest (rest (assoc non-terminal grammar))))

(define (random-element list)
  ;; returns a random top-level element from list
  (list-ref list (random (length list))))
```

If phrase is a list, like (NP VP),
then map generate over it
and append the results.

If a non-terminal, select
a random rewrite and
apply generate to it.

It's a terminal, so just
return a list with it as
the only element.

Parameterizing generate

- Let's change the package to not use global variables for grammar
- The *generate* function will take another parameter for the grammar and also pass it to *non-terminal?* and *rewrites*
- While we are at it, we'll make both parameters to *generate* optional with appropriate defaults

```
> (load "cfg2.ss")
> (generate)
(a table liked the blue robot)
> (generate grammar 'NP)
(the blue dog with a robot)
> (define g2 '(S -> (a S b) (a S b) (a S b) ()))
> (generate g2)
(a a a a a b b b b b b)
> (generate g2)
(a a a a a a a a a b b b b b b b b b b)
> (generate g2)
()
> (generate g2)
(a a b)
```

[cfg2.ss](#)
session

```
(define default-grammar '(S -> (NP VP) (NP VP) (NP VP) (NP VP) ...))
(define default-start 'S)
(define (generate (grammar default-grammar) (phrase default-start))
  ;; generate a random sentence or phrase from grammar
  (cond ((list? phrase)
        (apply append (map generate phrase)))
        ((non-terminal? phrase grammar)
         (generate grammar (random-element (rewrites phrase grammar))))
        (else (list phrase))))
(define (non-terminal? x grammar)
  ;; True iff x is a non-terminal in grammar
  (assoc x grammar))
(define (rewrites non-terminal grammar)
  ;; Return a list of the possible rewrites for non-terminal in grammar
  (rest (rest (assoc non-terminal grammar))))
```

Global variables
define defaults

optional
parameters

Pass value of
grammar to
subroutines

Subroutines
take new
parameter

```
(define default-grammar '(S -> (NP VP) (NP VP) (NP VP) (NP VP) ...))
(define default-start 'S)
(define (generate (grammar default-grammar) (phrase default-start))
  ;; generate a random sentence or phrase from grammar
  (cond ((list? phrase)
        (apply append (map generate phrase)))
        ((non-terminal? phrase grammar)
         (generate grammar (random-element (rewrites phrase grammar))))
        (else (list phrase))))
(define (non-terminal? x grammar)
  ;; True iff x is a non-terminal in grammar
  (assoc x grammar))
(define (rewrites non-terminal grammar)
  ;; Return a list of the possible rewrites for non-terminal in grammar
  (rest (rest (assoc non-terminal grammar))))
```

[cfg2.ss](#)

generate takes 2 args – we
want the 1st to be grammar's
current value and the 2nd to
come from the list

```
(define default-grammar '(S -> (NP VP) (NP VP) (NP VP) (NP VP) ...))
(define default-start 'S)
(define (generate (grammar default-grammar) (phrase default-start))
  ;; generate a random sentence or phrase from grammar
  (cond ((list? phrase)
        (apply append (map (curry generate grammar) phrase)))
        ((non-terminal? phrase grammar)
         (generate grammar (random-element (rewrites phrase grammar))))
        (else (list phrase))))
(define (non-terminal? x grammar)
  ;; True iff x is a non-terminal in grammar
  (assoc x grammar))
(define (rewrites non-terminal grammar)
  ;; Return a list of the possible rewrites for non-terminal in grammar
  (rest (rest (assoc non-terminal grammar))))
```

[cfg2.ss](#)

Curried functions

- Curried functions have lots of applications in programming language theory
- The curry operator is also a neat trick in our functional programming toolbox
- You can add them to Python and other languages, if the underlying language has the right support